Background Knowledge 1

Number

Contents:

A OPERATIONS WITH NUMBERS

There are four basic **operations** that are carried out with numbers:

- **Addition** to find a **sum**
  - When adding several numbers, we do not have to carry out the addition in the given order. Sometimes it is easier to change the order.
  - Adding zero does not change the value of a number.

For example: The sum of 3 and 16 is $3+16=19$.
The sum of 12 and 0 is $12+0=12$.
The sum of 187, 369, and 13 is

$$\begin{aligned} & 187+369+13 \\ &= \underbrace{187+13} + 369 \\ &= \quad 200 \quad + 369 \\ &= 569 \end{aligned}$$

- **Subtraction** to **take away** one number from another
  - To find the **difference** between two numbers, we *subtract* the smaller from the larger.
  - Subtracting zero does not change the value of a number.

For example: The difference between 37 and 82 is $82-37=45$.
12 take away zero is $12-0=12$.

- **Multiplication** to find a **product**
  - When multiplying several numbers, we do not have to carry out the multiplication in the given order.
  - Multiplying by one does not change the value of a number.
  - Multiplying by zero produces zero.

For example: The product of 3 and 5 is $3\times5=15$.
$3\times7\times2=2\times3\times7=42$
$1\times17=17\times1=17$
$0\times17=17\times0=0$

- **Division** to find a **quotient**
  - In the division $15\div3=5$, we say 15 is the **dividend** and 3 is the **divisor**.
  - Dividing by one does not change the value of a number.
  - Dividing by zero is meaningless. We say the result is **undefined**.

For example: $18\div1=18$
$0\div4=0$ but $4\div0$ is undefined.

EXERCISE 1A

1 Find:

a the sum of 4, 8, and 11

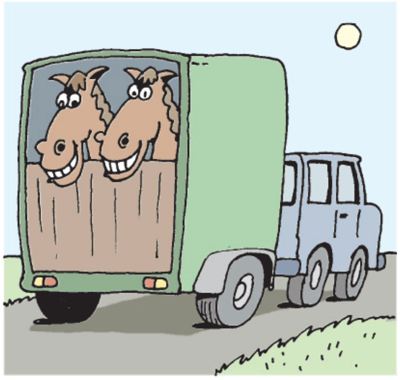
b the difference between 23 and 41

c the sum of the first 12 positive whole numbers

d by how much 407 exceeds 239.

2 **a** What number must be increased by 249 to get 752?

b What number must be decreased by 385 to get 2691?



3 Jose received €285 in wages whereas Juan received €312. How much more did Juan receive than Jose?

4 Emma's horse float has mass 406 kg and her two horses weigh 517 kg and 561 kg. Emma's car is allowed to tow 1500 kg. Is she allowed to transport both horses at the same time?

5 To help buy an apartment, Agneta borrowed \$26 200 from her parents. She has already paid them back amounts of \$515, \$872, and \$664. How much does Agneta still owe her parents?

6 Find:

a the product of 19 and 23

b the quotient of 1008 and 36

c the product of the first 6 positive whole numbers.

7 How many £3 buckets of chips must I sell to earn £246?

8 My orchard contains 8 rows of 12 apple trees. If each tree produces 400 fruit, how many apples can I harvest?

9 How many laps of a 400 m track does an athlete need to complete in a 10 000 m race?

10 An apartment complex has 6 buildings, each 28 storeys high, and on each storey there are 5 apartments.

a How many apartments are there in total?

b Each apartment owner has to pay \$3400 per year to maintain the buildings. What is the total annual budget for maintenance?

11 A cargo plane can carry 115 tonnes. How many plane loads are needed to transport 7245 tonnes of supplies?

B EXPONENT NOTATION

Rather than writing $6 \times 6 \times 6 \times 6$, we can write the same product as 6^4. We call this **exponent notation**.

We say that 6 is the **base** and 4 is the **exponent**, **power**, or **index**.

6^4 ← exponent, power, or index; 6 ← base

The following table shows the first five powers of 2.

Natural number	*Factorised form*	*Exponent form*	*Spoken form*
2	2	2^1	two
4	2×2	2^2	two squared
8	$2 \times 2 \times 2$	2^3	two cubed
16	$2 \times 2 \times 2 \times 2$	2^4	two to the fourth
32	$2 \times 2 \times 2 \times 2 \times 2$	2^5	two to the fifth

Any non-zero number raised to the power zero is equal to 1.

$$a^0 = 1, \ \ a \neq 0$$

0^0 is undefined.

Example 1 **Self Tutor**

a Write $3 \times 3 \times 3 \times 3 \times 8 \times 8$ in exponent form.

b Evaluate $2^3 \times 5^2 \times 7$.

a $3 \times 3 \times 3 \times 3 \times 8 \times 8 = 3^4 \times 8^2$

b
$$\begin{aligned} 2^3 \times 5^2 \times 7 &= \underbrace{2 \times 2 \times 2}_{8} \times \underbrace{5 \times 5}_{25} \times 7 \\ &= 8 \times 25 \times 7 \\ &= 200 \times 7 \\ &= 1400 \end{aligned}$$

EXERCISE 1B

1 Write in exponent form:

a 3×3
b $5 \times 5 \times 5$
c 7×7
d $2 \times 2 \times 2 \times 2 \times 2$
e $3 \times 3 \times 7 \times 7 \times 7$
f $2 \times 5 \times 5 \times 5$
g $3 \times 3 \times 3 \times 5 \times 11 \times 11$
h $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$
i $5 \times 2 \times 2 \times 5 \times 2$

2 Evaluate:

a 3^2
b $2^3 \times 3$
c $2^4 \times 5 \times 7$
d $2^2 \times 5^2 \times 13$
e $2 \times 5^2 \times 11$
f $2^3 \times 3^2 \times 5$

C FACTORS

The **factors** of a positive integer are the positive integers which divide exactly into it.

For example, the factors of 8 are 1, 2, 4, and 8 since $8 \div 1 = 8$
$8 \div 2 = 4$
$8 \div 4 = 2$
and $8 \div 8 = 1$.

3 is not a factor of 8 since $8 \div 3 = 2$ with remainder 2. We say that 8 is *not divisible* by 3.

All positive integers can be split into **factor pairs**.

For example: $8 = 1 \times 8$ or 2×4
$132 = 11 \times 12$

When we write a number as a product of factors, we say it is **factorised**.

10 may be factorised as a product of two factors in two ways: 1×10 or 2×5.

12 has factors 1, 2, 3, 4, 6, and 12. It can be factorised as a product of two factors in three ways: 1×12, 2×6, and 3×4.

EVEN AND ODD NUMBERS

A whole number is **even** if it has 2 as a factor and thus is divisible by 2.

A whole number is **odd** if it is not divisible by 2.

EXERCISE 1C

1 List *all* the factors of:

a 4 **b** 6 **c** 9 **d** 15
e 16 **f** 17 **g** 22 **h** 24
i 28 **j** 42 **k** 60 **l** 108

2 Complete the factorisations below:

a $36 = 6 \times ......$ **b** $38 = 2 \times ......$ **c** $48 = 12 \times ......$ **d** $90 = 5 \times ......$
e $88 = 8 \times ......$ **f** $54 = 3 \times ......$ **g** $72 = 12 \times ......$ **h** $60 = 12 \times ......$

3 List all factor pairs of:

a 10 **b** 14 **c** 18 **d** 20

4 **a** Beginning with 6, write three consecutive even numbers.
b Beginning with 11, write five consecutive odd numbers.

5 Use the words "even" and "odd" to complete these sentences:

a The sum of two even numbers is always ......
b The sum of two odd numbers is always ......
c The sum of three even numbers is always ......
d The sum of three odd numbers is always ......
e The sum of an odd number and an even number is always ......
f When an even number is subtracted from an odd number, the result is ......
g When an odd number is subtracted from an odd number, the result is ......
h The product of two odd numbers is always ......
i The product of an even and an odd number is always ......

D PRIMES AND COMPOSITES

A **prime** number is a natural number which has exactly two different factors.

A **composite** number is a natural number which has more than two factors.

For example: 11 is prime since it has only two factors: 1 and 11.

8 is composite since it has four factors: 1, 2, 4, and 8.

From the definition of prime and composite numbers we can see that:

The number 1 is neither prime nor composite.

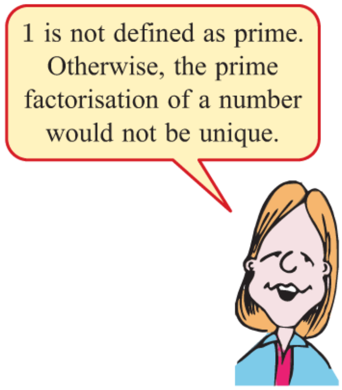
THE FUNDAMENTAL THEOREM OF ARITHMETIC

We can write 8 as the product 2×4, or as the product of *prime* factors $2 \times 2 \times 2$.

The **fundamental theorem of arithmetic** is:

> Every composite number can be written as the product of prime factors in exactly one way (ignoring order).

For example, we can write 252 as $2^2 \times 3^2 \times 7$ or $3^2 \times 7 \times 2^2$, but we cannot write it as a product of factors which includes any other prime.



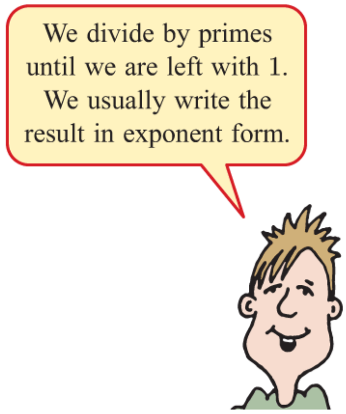
To express a composite number as the product of **prime numbers**, we systematically divide the number by the prime numbers which are its factors, starting with the smallest.

Example 2 — Self Tutor

Express 252 as the product of prime factors.

$$\begin{array}{r|r} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{aligned} \therefore \quad 252 &= 2 \times 2 \times 3 \times 3 \times 7 \\ &= 2^2 \times 3^2 \times 7 \end{aligned}$$



EXERCISE 1D

1 **a** List all the prime numbers less than 60.

b How many of these prime numbers are even? List them.

c Find:

i the smallest odd prime **ii** all odd two-digit composite numbers less than 30

iii a prime number whose two digits differ by 7.

2 Show that the following are composites by finding a factor other than 1 or itself:

a 985 **b** 7263 **c** 5840 **d** 1001

3 Express as a product of prime factors:

a 14 **b** 20 **c** 28 **d** 32 **e** 40

f 70 **g** 56 **h** 96 **i** 120 **j** 169

E HIGHEST COMMON FACTOR

> A number which is a factor of two or more other numbers is called a **common factor** of those numbers.

For example, 5 is a common factor of 25 and 35.

We can find the **highest common factor (HCF)** of two or more natural numbers by first expressing them as the product of prime factors.

Example 3 **Self Tutor**

Find the highest common factor (HCF) of 18 and 24.

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$18 = \mathbf{2} \times \mathbf{3} \times 3$
$24 = \mathbf{2} \times 2 \times 2 \times \mathbf{3}$

2×3 is common to the factorisations of both 18 and 24.

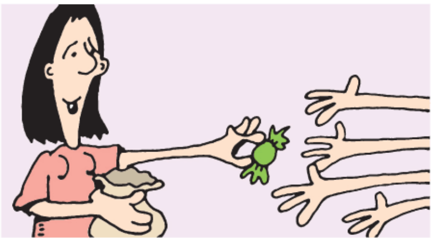
So, the highest common factor of 18 and 24 is $2 \times 3 = 6$.

EXERCISE 1E

1 Find the highest common factor of:

a 8 and 12 **b** 9 and 15 **c** 14 and 21 **d** 27 and 36
e 26 and 39 **f** 18 and 30 **g** 18, 24, and 45 **h** 32, 60, and 108

2 Alice has a packet containing 48 green lollies. Bob has a packet containing 56 red lollies. What is the highest number of friends, including Alice and Bob, that the lollies can be shared amongst so that each person receives the same number of green lollies, and each person receives the same number of red lollies?



F MULTIPLES

The **multiples** of any whole number have that number as a factor. They are obtained by multiplying the number by 1, then 2, then 3, then 4, and so on.

The multiples of 10 are 1×10, 2×10, 3×10, 4×10, 5×10, ...
or 10, 20, 30, 40, 50, ...

Likewise, the multiples of 15 are 15, 30, 45, 60, 75, ...

The number 30 is a multiple of both 10 and 15, so we say 30 is a **common multiple** of 10 and 15.

Example 4 **Self Tutor**

Find common multiples of 4 and 6 between 20 and 40.

The multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, 40, ...

The multiples of 6 are 6, **12**, 18, **24**, 30, **36**, 42, ...

$\therefore$ the common multiples between 20 and 40 are 24 and 36.

The **lowest common multiple** (**LCM**) of two or more numbers is the smallest number which is a multiple of *each* of those numbers.

Example 5 **Self Tutor**

Find the lowest common multiple of 9 and 12.

The multiples of 9 are: 9, 18, 27, **36**, 45, 54, 63, **72**, 81, ...

The multiples of 12 are: 12, 24, **36**, 48, 60, **72**, 84, ...

$\therefore$ the LCM is 36.

EXERCISE 1F

1. List the first six multiples of:
   a. 4
   b. 5
   c. 7
   d. 11
2. Find the:
   a. fourth multiple of 6
   b. sixth multiple of 9.
3. List the numbers from 1 to 40.
   a. Put a circle around each multiple of 3.
   b. Put a square around each multiple of 5.
   c. List the common multiples of 3 and 5 which are less than 40.
4. The first ten multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120.
   State the numbers from the list which are common multiples of:
   a. 9 and 12
   b. 12 and 15
   c. 9, 12, and 15.
5. Find the lowest common multiple of:
   a. 2 and 5
   b. 3 and 7
   c. 4 and 5
   d. 6 and 8
   e. 6 and 9
   f. 10 and 12
   g. 4, 5, and 7
   h. 6, 9, and 12.
6. Find:
   a. the smallest multiple of 7 that is greater than 100
   b. the greatest multiple of 9 that is less than 200.
7. Three clocks start chiming at exactly the same instant. One chimes every 3 hours, one every 4 hours, and the other every 6 hours. When will they next chime together?
8. The football fields at three different schools were measured, and it was found that their perimeters were 320 m, 360 m, and 400 m. The students at each school are to run the same distance, and this must be a whole number of laps. What is the shortest distance that they need to run?

G SQUARE ROOTS AND CUBE ROOTS

SQUARE ROOTS

The **square root** of the number a is the positive number which, when squared, gives a.

We write the square root of a as $\sqrt{a}$.

$$(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a$$

For example: since $4^2 = 16$, $\sqrt{16} = 4$
since $5^2 = 25$, $\sqrt{25} = 5$.

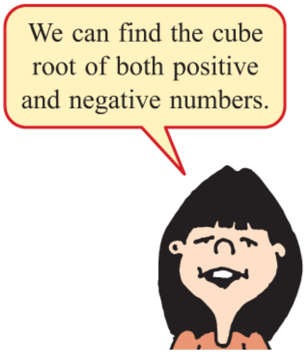
If a number is not a perfect square, then its square root will not be a whole number. In this case, we can estimate the square root of the number by considering the square numbers either side of it. For example, 30 is between $5^2 = 25$ and $6^2 = 36$, so $\sqrt{30}$ is between 5 and 6.

CUBE ROOTS

The **cube root** of a number a is the number which, when cubed, gives a.

We write the cube root of a as $\sqrt[3]{a}$.

$$(\sqrt[3]{a})^3 = \sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$$



For example: since $2^3 = 8$, $\sqrt[3]{8} = 2$
since $3^3 = 27$, $\sqrt[3]{27} = 3$
since $(-3)^3 = -27$, $\sqrt[3]{27} = -3$

EXERCISE 1G

1 Without using your calculator, find:

a $\sqrt{49}$ **b** $\sqrt{64}$ **c** $\sqrt{121}$ **d** $\sqrt{144}$

2 Use your calculator to find:

a $\sqrt{289}$ **b** $\sqrt{576}$ **c** $\sqrt{1521}$ **d** $\sqrt{2304}$

3 Between which two consecutive integers do the following values lie?

a $\sqrt{12}$ **b** $\sqrt{28}$ **c** $\sqrt{103}$ **d** $\sqrt{150}$

Check your answers using a calculator.

4 Without using your calculator, find:

a $\sqrt[3]{1}$ **b** $\sqrt[3]{-1}$ **c** $\sqrt[3]{-27}$ **d** $\sqrt[3]{8}$ **e** $\sqrt[3]{-8}$

f $\sqrt[3]{-64}$ **g** $\sqrt[3]{125}$ **h** $\sqrt[3]{216}$ **i** $\sqrt[3]{-1000}$ **j** $\sqrt[3]{343}$

H ORDER OF OPERATIONS

When two or more operations are carried out, different answers can result depending on the **order** in which the operations are performed.

For example, consider the expression $11 - 4 \times 2$.

Bruce decided to subtract first, then multiply:

$$\begin{aligned} &\underbrace{11 - 4} \times 2 \\ =\ & \quad 7 \quad \times 2 \\ =\ & 14 \end{aligned}$$

Poj decided to multiply first, then subtract:

$$\begin{aligned} &11 - \underbrace{4 \times 2} \\ =\ & 11 - \quad 8 \\ =\ & 3 \end{aligned}$$

Which answer is correct, 14 or 3?

To avoid this problem, a set of rules for the order of performing operations has been agreed upon by all mathematicians.

RULES FOR ORDER OF OPERATIONS

- Perform operations within **B**rackets first.
- Calculate any part involving **E**xponents.
- Starting from the left, perform all **D**ivisions and **M**ultiplications as you come to them.
- Finally, working from the left, perform all **A**dditions and **S**ubtractions.

The word **BEDMAS** may help you remember this order.

In addition, we agree that:

- If an expression contains more than one set of brackets, evaluate the innermost brackets first.
- In a fraction, we assume that the numerator and denominator each have brackets around them. We must evaluate them first before doing the division.

Using these rules, Poj's method is correct in the above example, and $11 - 4 \times 2 = 3$.

Example 6 Self Tutor

Evaluate: $35 - 10 \div 2 \times 5 + 3$

$35 - 10 \div 2 \times 5 + 3$
$= 35 - 5 \times 5 + 3$ {division and multiplication working from left}
$= 35 - 25 + 3$
$= 10 + 3$ {subtraction and addition working from left}
$= 13$

EXERCISE 1H

1 Evaluate:

a $6 - 3 + 4$	**b** $7 \times 4 \div 2$	**c** $3 + 2 \times 5$	**d** $3 \times 2 - 1$
e $16 \div 4 \times 2$	**f** $15 \div 5 + 2$	**g** $9 - 6 \div 3$	**h** $4 + 7 - 3 \times 2$
i $3 \times 4 - 2 \times 5$	**j** $3 + 9 \div 3 - 2$	**k** $7 - 9 \div 3 \times 2$	**l** $13 - 2 \times 6 + 7$

Example 7 Self Tutor

Evaluate: $2 \times (3 \times 6 - 4) + 7$

$2 \times (3 \times 6 - 4) + 7$
$= 2 \times (18 - 4) + 7$ {inside brackets, multiply}
$= 2 \times 14 + 7$ {evaluate expression in brackets}
$= 28 + 7$ {multiplication next}
$= 35$ {addition last}

If you do not follow the order rules, you are likely to get the wrong answer.

2 Evaluate:

a $(11-6)\times 3$ **b** $9\div(7-4)$ **c** $(2+7)\div 3$

d $4\times(6-2)$ **e** $7+(2+3)\times 5$ **f** $18\div(1+5)-1$

g $2+3\times(7-2)$ **h** $3+(17-8)\div 9$ **i** $4\times 3-(6-2)$

j $4\div(3-1)+6$ **k** $(7+11)\div(7-4)$ **l** $(4-1)\times(7+5)$

m $2\times(3-4)+(7-1)$ **n** $(14-3\times 2)\div(7-3)$ **o** $(22-3\times 5)\times(8-3\times 2)$

Example 8 Self Tutor

Evaluate: $5+[13-(8\div 4)]$

$$\begin{aligned} & 5+[13-(8\div 4)] \\ &= 5+[13-2] && \{\text{innermost brackets first}\} \\ &= 5+11 && \{\text{remaining brackets next}\} \\ &= 16 && \{\text{addition last}\} \end{aligned}$$

Evaluate the innermost brackets first.

3 Evaluate:

a $3\times[2+(7-5)]$ **b** $3+[2\times(7-5)]$

c $[(13-7)\div 2]+11$ **d** $[14\div(2+5)]\times 3$

e $3+[32\div(2+6)]\div 2$ **f** $3\times[(32\div 2)+6]-2$

Example 9 Self Tutor

Evaluate: $\dfrac{16-(4-2)}{14\div(3+4)}$

$$\begin{aligned} & \frac{16-(4-2)}{14\div(3+4)} \\ &= \frac{16-2}{14\div 7} && \{\text{brackets first}\} \\ &= \frac{14}{2} && \{\text{evaluate numerator, denominator}\} \\ &= 7 && \{\text{do the division}\} \end{aligned}$$

For a fraction, we evaluate the numerator and denominator separately, then perform the division.

4 Evaluate:

a $\dfrac{19-3}{2}$ **b** $\dfrac{11-6}{4\times 5}$ **c** $\dfrac{6\times(7-2)}{10}$ **d** $\dfrac{18-2\times 7}{6\div 3}$

5 Evaluate:

a $3+5^2$ **b** 7^2-18 **c** $5^2-6\times 2$

d $(13-4)\div 3^2$ **e** $48\div(5-3)^2$ **f** $2\times 3^3-(11-7)^2$

g $2^3+3\times 5$ **h** $5^2-2^3\times 3$ **i** $3^2-2\times 5^0$

j $\sqrt{9}-\sqrt{4}$ **k** $-2+2\sqrt{16}$ **l** $3\sqrt{4}-(\sqrt{7})^2$

6 Evaluate:

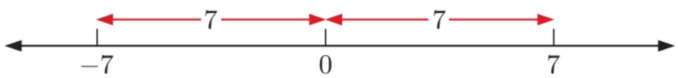
a $3 \times -2 + 18$
b $-3 \times -2 - 18$
c $23 - 5 \times -3$
d $[3 - (-2 + 7)] + 4$
e $(18 \div 3) \times -2$
f $2(7 - 13) - (6 - 12)$
g $-6 \times (2 - 7)$
h $-(14 - 8) \div -2$
i $-18 - (8 - 15)$
j $-52 \div (6 - 19)$
k $\dfrac{38 - -4}{6 \times -7}$
l $\dfrac{28 - (-3 \times 4)}{10 \times -2}$
m $\sqrt{35 - 2 \times 5}$
n $\sqrt[3]{64} - 2^3$
o $\dfrac{-6 + \sqrt{6^2 - 4 \times 8}}{4}$

I MODULUS OR ABSOLUTE VALUE

The **modulus** or **absolute value** of a real number is its size, ignoring its sign.

The modulus of a real number is its *distance* from zero on the number line. Because the modulus is a distance, it cannot be negative.

For example: the modulus of 7 is 7, and
the modulus of -7 is also 7.



EXERCISE 1I

1 Find the value of:

a $|-3|$
b $|4| + |-7|$
c $|-2| - |-9|$
d $5 - (-11)$
e $|5| - |-11|$
f $|5 - (-11)|$
g $\left|(-2)^2 + 11(-2)\right|$
h $|-6| - |-8|$
i $|-6 - (-8)|$

2 Find the value of:

a $\left|\dfrac{4}{-2}\right|$
b $\dfrac{|4|}{|-2|}$
c $\left|\dfrac{-3}{-12}\right|$
d $\dfrac{|-3|}{|-12|}$

J ROUNDING NUMBERS

There are many occasions when it is sensible to give an **approximate** answer.

For example, it is unreasonable to give the exact distance between the Earth and the sun, because it is continually changing. The distance varies from its *perihelion*, about 147 million km, to its *aphelion*, about 152 million km.

We use the symbol $\approx$ or sometimes $\doteqdot$ to show that an answer has been approximated.

RULES FOR ROUNDING OFF

- If the digit after the one being rounded off is **less than 5** (0, 1, 2, 3, or 4) we round **down**.
- If the digit after the one being rounded off is **5 or more** (5, 6, 7, 8, 9) we round **up**.

Example 10 Self Tutor

Round off: **a** 436 to the nearest 10 **b** 716 to the nearest 100
c 1050 to the nearest 100 **d** 19 628 to the nearest 1000

a $436 \approx 440$ {round up, as 6 is greater than 5}
b $716 \approx 700$ {round down, as 1 is less than 5}
c $1050 \approx 1100$ {5 is rounded up}
d $19\,628 \approx 20\,000$ {round up, as 6 is greater than 5}

EXERCISE 1J.1

1 Round off to the nearest 10:

a 86 **b** 81 **c** 85 **d** 128 **e** 162
f 104 **g** 635 **h** 1822 **i** 699 **j** 3045

2 Round off to the nearest 100:

a 215 **b** 264 **c** 3750 **d** 3950 **e** 26 341

3 Round off to the nearest 1000:

a 8365 **b** 3500 **c** 19 210 **d** 19 650 **e** 114 823

4 Round off to the accuracy given:

a The height of Mount Everest is 8848 m. (to the nearest 10 m)
b The surface area of Lake Baikal in Russia is 31 722 km^2. (to the nearest 1000 km^2)
c The population of New Zealand is 4 749 598. (to the nearest 10 000)
d The attendance at an English football match was 85 512 people. (to the nearest 100 people)
e The area of Australia is 7 692 024 km^2. (to the nearest 100 000 km^2)
f An average weight of an adult African elephant is 5443 kg. (to the nearest 100 kg)
g The distance between Paris and Sydney is 16 950 km. (to the nearest 100 km)
h The distance from Earth to the moon at perigee is 363 104 km. (to the nearest 100 000 km)
i The population of South America in 2018 was 428 240 515. (to the nearest 1 000 000)

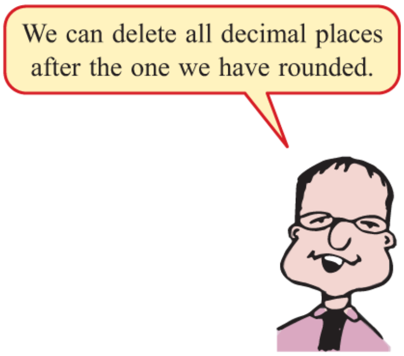
ROUNDING DECIMAL NUMBERS

A survey found that a total of 6428 flights were taken by 825 people last year. It is not sensible to give the average number of flights per person as 7.791 515 152 . An approximate answer of 7.8 is more appropriate.

Example 11 Self Tutor

Round:
a 8.43 to one decimal place
b 3.5169 to two decimal places.

a $8.43 \approx 8.4$ {round down, as $3 < 5$}
b $3.5169 \approx 3.52$ {round up, as $6 > 5$}



EXERCISE 1J.2

1 Round the following to the number of decimal places stated in brackets:

a 6.181 [1] **b** 6.181 [2] **c** 3.25 [1] **d** 17.403 [2]

e 2.131 58 [3] **f** 0.1940 [1] **g** 0.0972 [2] **h** 102.382 [2]

2 In 2009 Usain Bolt ran 100 m in 9.58 seconds. Round this time to 1 decimal place.

3 The average height of children in a class was 1.435 m. Round this height to 2 decimal places.

4 The thickness of a sheet of paper is 0.012 cm. Round this thickness to 2 decimal places.

5 The number pi is a mathematical constant. The first six digits of pi are 3.141 59.
Round pi to: **a** 1 decimal place **b** 3 decimal places **c** 4 decimal places.

6 The fraction $\frac{5}{19} \approx 0.263\,157\,895$. Round this number to:

a 1 decimal place **b** 2 decimal places **c** 6 decimal places.

7 Evaluate, giving your answer to 3 decimal places:

a $\sqrt{2}$ **b** $\sqrt{5}$ **c** $\sqrt{23}$ **d** $\sqrt[3]{4}$ **e** $\sqrt[3]{-15}$ **f** $\sqrt[3]{450}$

8 Calculate the following, rounding your answers to 2 decimal places:

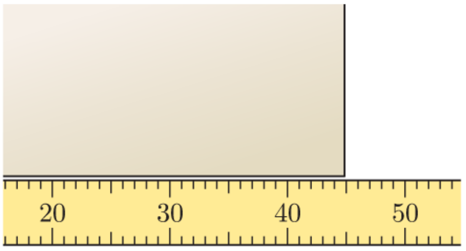
a $(16.8 + 12.4) \times 17.1$ **b** $16.8 + 12.4 \times 17.1$ **c** $127 \div 9 - 5$

d $127 \div (9 - 5)$ **e** $37.4 - 16.1 \div (4.2 - 2.7)$ **f** $\dfrac{16.84}{7.9 + 11.2}$

g $\dfrac{27.4}{3.2} - \dfrac{18.6}{16.1}$ **h** $\dfrac{27.9 - 17.3}{8.6} + 4.7$ **i** $\dfrac{0.0768 + 7.1}{18.69 - 3.824}$

9 Over a 23 game water polo season, Kerry scored 40 goals for her team. Find Kerry's average number of goals per game, correct to 2 decimal places.

10 Wang used a tape measure to check the length of his kitchen bench. After viewing the tape, he recorded the length as 2.45 m, which he rounded up to 2.5 m.
When his mother asked him about the length, he said it was about 3 m.
Explain what Wang has done wrong, and discuss why we need to be careful when we make approximations.



ROUNDING OFF TO SIGNIFICANT FIGURES

The first **significant figure** of a decimal number is the first (left-most) non-zero digit.

For example:
- the first significant figure of 4567 is 4
- the first significant figure of 0.012 34 is 1.

Every digit to the right of the first significant figure is regarded as another significant figure.

To round off to a number of significant figures:

Count off the specified number of significant figures then look at the next digit.
- If the digit is less than 5, round **down**.
- If the digit is 5 or more, round **up**.

Delete all figures following the significant figures, replacing with 0s where necessary.

Example 12 **Self Tutor**

Round: **a** 3.461 to 2 significant figures **b** 0.007 24 to 2 significant figures
c 708 to 1 significant figure **d** 20.158 to 3 significant figures.

a $3.461 \approx 3.5$ {2 significant figures}

This is the 2nd significant figure, so we look at the next digit which is 6.
The 6 tells us to round the 4 up to a 5 and delete the remaining digits.

b $0.007\,24 \approx 0.0072$ {2 significant figures}

These zeros at the front are place holders and so must stay.
The first significant figure is 7, and the second significant figure is 2.
The 4 tells us to leave the 2 as it is and delete the remaining digits.

c $708 \approx 700$ {1 significant figure}

7 is the first significant figure so it has to be rounded.
The 0 tells us to keep the original 7 in the hundreds place, so we convert the 08 into 00.
These two zeros are place holders. They are not significant figures, but they need to be there to make sure the 7 has value 700.

d $20.158 \approx 20.2$ {3 significant figures}

This 0 is significant as it lies between two non-zero digits.
The third significant figure is 1.
The 5 tells us to round the 1 up to a 2 and delete the remaining digits.

In IB examinations you are expected to give answers to 3 significant figures unless otherwise specified in the question.

EXERCISE 1J.3

1 Round to 2 significant figures:

a 128 **b** 8342 **c** 2.568 **d** 0.0134
e 163 870 **f** 1.086 **g** 3958 **h** 6.611

2 Round to 3 significant figures:

a 83 064 **b** 10 044 **c** 0.105 26 **d** 31.695
e 70.707 **f** 4.0007 **g** 0.036 71 **h** 19.989

3 Round to 4 significant figures:

a 16.382 **b** 438.207 **c** 6 873 681 **d** 0.028 885

4 The exact crowd size at a rock concert was 96 257 people. Round the crowd size to:

a 1 significant figure **b** 2 significant figures **c** 3 significant figures.

5 Evaluate the following, giving your answers to 3 significant figures:

a $\sqrt{7}$ **b** 2π **c** $36 \div 17$ **d** 517×3802
e $(0.986)^5$ **f** $\dfrac{16.3 - 2.68}{3.1}$ **g** $\sqrt{5.4 - 2.18}$ **h** $\dfrac{9.58}{\sqrt{2.8}}$

6 A theatre has 32 rows with 28 seats in each. Find the total number of seats, rounding your answer to 2 significant figures.

7 A ballroom has dimensions $30.1\text{ m} \times 8.5\text{ m}$. Find its area, rounding your answer to 3 significant figures.

8 The proceeds of a garage sale were $752.25, and this was shared equally between 4 people. Calculate the amount each person received:

a to 3 significant figures **b** to 5 significant figures.

K APPROXIMATIONS

A fast way of estimating a calculation is to perform a **one figure approximation**:

- Leave single digit numbers as they are.
- Round all other numbers to one significant figure.
- Perform the calculation.

Example 13 Self Tutor

Estimate: **a** 872×52 **b** $61\,812 \div 384$ **c** 4.37×0.482

a 872×52
$\approx 900 \times 50$
$\approx 45\,000$

b $61\,812 \div 384$
$\approx 60\,000 \div 400$
$\approx 600 \div 4$
≈ 150

c 4.37×0.482
$\approx 4 \times 0.5$
≈ 2

DEMO

EXERCISE 1K

1 Estimate using a one figure approximation:

a 32×6 **b** 58×7 **c** 81×30 **d** 207×3 **e** 487×50
f 6117×4 **g** 48×23 **h** 61×42 **i** 103×47 **j** 3125×18
k 422×307 **l** 3818×27 **m** 2.7×1.15 **n** 5.36×0.68 **o** 28.37×6.13

2 Estimate using a one figure approximation:

a $86 \div 3$ **b** $64 \div 5$ **c** $512 \div 21$ **d** $610 \div 43$
e $4182 \div 19$ **f** $78\,638 \div 82$ **g** $318 \div 62$ **h** $47\,320 \div 193$
i $0.628 \div 3$ **j** $46.1 \div 5.2$ **k** $631.7 \div 0.29$ **l** $18.7 \div 3.86$

3 Estimate using one figure approximations:

a the cost of 8 kg of apples at €2.80/kg **b** the cost of 3 airline tickets at $213 each
c the cost of 7 theatre tickets at $87.30 each **d** the cost of 55 litres of fuel at £1.49/litre.

4 Estimate using one figure approximations:

a the distance travelled if Brodie drives for 4.2 hours at 63 km h^{-1}
b the number of days in 14 years
c the average weight carried per truck if 423 tonnes of cargo is divided equally between 18 trucks.

Background Knowledge

2

Algebra

Contents:

We use **algebra** to write mathematical ideas in a convenient way. In algebra we use letters or symbols to represent unknown quantities called *variables*, whose values can vary depending on the situation.

You should be familiar with these important words associated with algebra:

- $2x + 3$ is an **expression** for the quantity which is three more than twice x.
- $2x+3=5$ is an **equation** which says that the quantity $2x+3$ has the value 5. We can **solve** the equation to find the value of x.
- $2x+3>5$ is an **inequality** or **inequation** which says that the value of $2x+3$ is more than 5.
- $y = 2x+3$ is a **formula** which connects the two **variables** x and y. If we know the value of one of the variables then we can **substitute** this value to determine the other variable. We can also **rearrange** formulae to write them in more convenient forms.

A ALGEBRAIC NOTATION

In algebra we agree:

- to **leave out** the "$\times$" signs between multiplied quantities
- to write **numerals (numbers) first** in any product
- where products contain two or more letters, we write them in **alphabetical order**.

For example:
- $3b$ is used rather than $3\times b$ or $b3$
- $3bc$ is used rather than $3cb$.

Example 1 Self Tutor

Write in product notation:

a $t\times 6s$ **b** $4\times k+m\times 3$ **c** $3\times(r+s)$

a $t\times 6s$
$= 6st$

b $4\times k+m\times 3$
$= 4k+3m$

c $3\times(r+s)$
$= 3(r+s)$

WRITING SUMS AS PRODUCTS

Sums of identical terms can be written using product notation.

For example, $3+3+3+3 = 4\times 3$ {4 lots of 3}

$\therefore\ b+b+b+b = 4\times b = 4b$ {4 lots of b}

Example 2 Self Tutor

Simplify: **a** $r+r+r+s+s$ **b** $d+d-(a+a+a+a)$

a $r+r+r+s+s$
$= 3r+2s$

b $d+d-(a+a+a+a)$
$= 2d-4a$

EXPONENT NOTATION

We use **exponent** or **index notation** to simplify algebraic expressions in the same way as we did for numbers.

For example, $3 \times 3 \times 3 \times 3 = 3^4$ and $b \times b \times b \times b = b^4$.

Example 3 — Self Tutor

Simplify: **a** $8 \times b \times b \times a \times a \times a$ **b** $k + k - 3 \times d \times d \times d$

a $8 \times b \times b \times a \times a \times a$
$= 8a^3b^2$

b $k + k - 3 \times d \times d \times d$
$= 2k - 3d^3$

EXERCISE 2A

1 Simplify using product notation:

a $5 \times x$ **b** $c \times 2$ **c** $q \times 7$
d $f \times 4g$ **e** $6q \times p$ **f** $r \times 9s$
g $2a \times 3b$ **h** $m \times 4n$ **i** $a \times 5 \times b$
j $q \times 2 \times p$ **k** $j \times k \times l$ **l** $p \times h \times d$

2 Simplify:

a $p \times q + r$ **b** $4 \times x + 5 \times y$ **c** $2 \times a - b$
d $b \times a - c$ **e** $b - a \times c$ **f** $f - g \times 7$
g $c \times a + d \times a$ **h** $12 - r \times s \times 6$ **i** $3 \times (x + y)$
j $5 \times (d - 1)$ **k** $(w - x) \times 8$ **l** $p \times q \times (r - 2)$

3 Simplify using product notation:

a $b + b$ **b** $q + q + q$ **c** $x + x + y + y + y + y$
d $c + c + c + e$ **e** $3 + z + y + y$ **f** $a + a + a + a + 7$
g $g + g + 2 + g + g$ **h** $3 - (d + d + d)$ **i** $s - t + t$
j $s - (t + t)$ **k** $4 + r + r + r + 1$ **l** $2 + a + a + b + b$

4 Write in expanded form:

a a^4 **b** f^2 **c** $5a^2$
d $(5a)^2$ **e** $4p^3$ **f** $(4p)^3$
g $3t^5$ **h** $5x^2y$ **i** $5(xy)^2$
j $7f^2g^3$ **k** $p^2 + 2q$ **l** $p^3 - 3q^2$

We use exponents and product notation to simplify the look of an expression.

5 Write in simplest form:

a $3 \times k \times k$ **b** $4 \times a \times a \times a$ **c** $2 \times d \times d \times d \times d$
d $4 \times p \times q \times q$ **e** $3 \times f \times g \times f \times g$ **f** $w \times w \times x \times y \times y \times y$
g $m + m \times m$ **h** $n \times n \times n + n$ **i** $y \times y - z \times z \times z \times z$
j $a \times a + 7 \times a$ **k** $8 \times b - b \times b \times b$
l $2 \times p \times q \times q + 6 \times r \times r \times s$ **m** $h \times h \times 2 - h \times j$
n $3 \times x + 5 \times x \times x \times x$ **o** $a \times a + 2 \times b \times b \times b - a \times b \times b$

B COLLECTING LIKE TERMS

In algebra, **like terms** are terms which contain the same variables to the same exponent.

For example:

- $2xy$ and $-5xy$ are like terms
- a^2 and $-3a$ are unlike terms because the exponents of a are not the same.

Algebraic expressions can often be simplified by adding or subtracting like terms. This is sometimes called **collecting like terms**.

Consider $2a + 4a = \underbrace{a + a}_{\text{“2 lots of } a\text{”}} + \underbrace{a + a + a + a}_{\text{“4 lots of } a\text{”}}.$

In total we have 6 lots of a, so $2a + 4a = 6a$.

Example 4 — Self Tutor

Simplify, where possible, by collecting like terms:

a $3x + 2x$ **b** $7a - 3a$ **c** $-2x + 3 - x$
d $3bc + bc$ **e** $2x - x^2$

a $3x + 2x$
$= 5x$

b $7a - 3a$
$= 4a$

c $-2x + 3 - x$
$= -3x + 3$
$\{-2x$ and $-x$ are like terms$\}$

d $3bc + bc$
$= 4bc$

e $2x - x^2$ is in simplest form
$\{2x$ and $-x^2$ are unlike terms$\}$

EXERCISE 2B

1 Simplify, where possible, by collecting like terms:

a $2 + x + 4$ **b** $q + 5 + 6$ **c** $b + 3 + b + b$
d $a + a + 7$ **e** $d + d$ **f** $q + 1 + q + 4$
g $5y - 3y$ **h** $4z - z$ **i** $g^2 + g^2$
j $5x + 5$ **k** $5w^2 - 4w^2$ **l** $3x - 3x^2$
m $3x - x$ **n** $3ab + 6ab$ **o** $m + m + m + m$

2 Simplify, where possible:

a $8p - 8p$ **b** $8p - p$ **c** $8p - 8$
d $7pq - pq$ **e** $ab + 3ab$ **f** $3q^2 - q^2$
g $3w + 4w + 5w$ **h** $8xy + 5yx$ **i** $2z + 5z - 4z$
j $2m + 3m - 5m$ **k** $5d + 4d - 9$ **l** $3g + 4g - 7g^2$
m $s + 3s + 4s^2$ **n** $2x^2 + 2x + 2$ **o** $2a^2 - b^2$

3 Simplify, where possible:

a $7x + x$	**b** $7x - x$	**c** $-7x - x$	**d** $p^2 + 2p$
e $k + 4k - 5$	**f** $n - 6n - 5n$	**g** $-11m - 4m$	**h** $4j - 9j + 4$
i $-y - -8y$	**j** $8y - y$	**k** $8y - -y$	**l** $y - -4y$

Example 5 Self Tutor

Simplify, by collecting like terms:

a $2 + 3a - 3 - 2a$ **b** $x^2 - 2x + 3x - 2x^2$

a
$$\begin{aligned} &2 + 3a - 3 - 2a \\ &= 3a - 2a + 2 - 3 \\ &= a - 1 \end{aligned}$$
{$3a$ and $-2a$ are like terms, 2 and -3 are like terms}

b
$$\begin{aligned} &x^2 - 2x + 3x - 2x^2 \\ &= x^2 - 2x^2 - 2x + 3x \\ &= -x^2 + x \end{aligned}$$
{x^2 and $-2x^2$ are like terms, $-2x$ and $3x$ are like terms}

4 Simplify:

a $x + 5 - 3x - 6$	**b** $8t + 4 - 3t - 1$	**c** $x + 5y - 6y - 3x$
d $pq + 3 + 5pq - 7$	**e** $-cd + 2cd + 9cd$	**f** $2a - 6 + 6 - 3a$
g $12x^2 + 5 - 7x^2 - 7$	**h** $-5n + 3 + 2n - 6$	**i** $2v - 7v + w - 6w$
j $-3x^3 - 2x^2 + 3x^3 - x^2$	**k** $4a - 3b - -a - 4b$	**l** $-2z - 3 - 3z - 4$
m $2p + pq - 3pq - p$	**n** $-6mn + 3m - mn - 5m$	

C EXPANSION AND SIMPLIFICATION

We use the following rules to expand brackets in algebraic expressions:

- $\boldsymbol{a(b + c) = ab + ac}$ **{distributive law}**
- $\boldsymbol{(a + b)(c + d) = ac + ad + bc + bd}$ **{FOIL rule}**
- $\boldsymbol{(a + b)(a - b) = a^2 - b^2}$ **{difference between two squares}**
- $\boldsymbol{(a + b)^2 = a^2 + 2ab + b^2}$ **{perfect squares}**

Once an algebraic expression has been expanded, it can be simplified by collecting like terms.

Example 6 Self Tutor

Expand and simplify:

a $3(x + 4) + 2x$ **b** $(3x - 2)(x + 3)$

a $3(x + 4) + 2x = 3x + 12 + 2x$ {distributive law}
$= 5x + 12$ {collecting like terms}

b $(3x - 2)(x + 3) = 3x^2 + 9x - 2x - 6$ {FOIL rule}
$= 3x^2 + 7x - 6$ {collecting like terms}

EXERCISE 2C

1 Expand and simplify using the distributive law $a(b+c)=ab+ac$:

- **a** $2(x+4)$
- **b** $3(x-1)$
- **c** $-2(x+2)$
- **d** $-(2-x)$
- **e** $x(x+1)$
- **f** $5(x+4)+3x$
- **g** $4(x-2)+7$
- **h** $x(x-2)+x$
- **i** $x(3-x)+x^2$
- **j** $3(2x-1)+5$
- **k** $6-2(3x-5)$
- **l** $3(2x+5)+4(5+4x)$
- **m** $2x(x+1)-3x$
- **n** $2(a-3)-a(2-a)$
- **o** $5(2a-3b)-6(a-2b)$

2 Expand and simplify using the rule $(a+b)(c+d)=ac+ad+bc+bd$:

- **a** $(x+2)(x-3)$
- **b** $(x-1)(x+4)$
- **c** $(x+3)(x+2)$
- **d** $(2x+3)(x+1)$
- **e** $(3x+4)(x+2)$
- **f** $(5x-2)(2x+1)$
- **g** $(x+2)(3x-5)$
- **h** $(7-2x)(2+3x)$
- **i** $(1-3x)(5+2x)$
- **j** $(3x+4)(5x-3)$
- **k** $(1-3x)(2-5x)$
- **l** $(7-x)(3-2x)$
- **m** $(5-2x)(3-2x)$
- **n** $-(x+1)(x+2)$
- **o** $-2(x-1)(2x+3)$

3 Expand using the difference between two squares rule $(a+b)(a-b)=a^2-b^2$:

- **a** $(x+6)(x-6)$
- **b** $(x+8)(x-8)$
- **c** $(2x-1)(2x+1)$
- **d** $(3x-2)(3x+2)$
- **e** $(4x+5)(4x-5)$
- **f** $(5x-3)(5x+3)$
- **g** $(3-x)(3+x)$
- **h** $(7-x)(7+x)$
- **i** $(7+2x)(7-2x)$
- **j** $\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)$
- **k** $\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)$
- **l** $\left(2x-\sqrt{3}\right)\left(2x+\sqrt{3}\right)$

4 Expand and simplify using the perfect square expansion rule $(a+b)^2=a^2+2ab+b^2$:

- **a** $(x+5)^2$
- **b** $(x+7)^2$
- **c** $(x-2)^2$
- **d** $(x-6)^2$
- **e** $(3+x)^2$
- **f** $(5+x)^2$
- **g** $(11-x)^2$
- **h** $(10-x)^2$
- **i** $(2x+7)^2$
- **j** $(3x+2)^2$
- **k** $(5-2x)^2$
- **l** $(7-3x)^2$

D ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

To add or subtract algebraic fractions, we combine them into a single fraction with the **least common denominator** (LCD).

Example 7 **Self Tutor**

Write as a single fraction:

a $2+\frac{3}{x}$ **b** $\frac{x-1}{3}-\frac{x+3}{2}$

a
$$\begin{aligned} & 2+\frac{3}{x} \\ &= 2\left(\frac{x}{x}\right)+\frac{3}{x} \quad \{\text{LCD}=x\} \\ &= \frac{2x+3}{x} \end{aligned}$$

b
$$\begin{aligned} & \frac{x-1}{3}-\frac{x+3}{2} \\ &= \frac{2}{2}\left(\frac{x-1}{3}\right)-\frac{3}{3}\left(\frac{x+3}{2}\right) \quad \{\text{LCD}=6\} \\ &= \frac{2(x-1)-3(x+3)}{6} \\ &= \frac{2x-2-3x-9}{6} \\ &= \frac{-x-11}{6} \end{aligned}$$

EXERCISE 2D

1 Write as a single fraction:

a $3+\frac{x}{5}$ **b** $1+\frac{3}{x}$ **c** $3+\frac{x-2}{2}$ **d** $3-\frac{x-2}{4}$

e $\frac{2+x}{3}+\frac{x-4}{5}$ **f** $\frac{2x+5}{4}-\frac{x-1}{6}$ **g** $\frac{2}{x}+\frac{x}{2}$ **h** $\frac{x+2}{3}-\frac{x+1}{x}$

Example 8 **Self Tutor**

Write $\frac{3x+1}{x-2}-2$ as a single fraction.

$$\begin{aligned} \frac{3x+1}{x-2}-2 &= \left(\frac{3x+1}{x-2}\right)-2\left(\frac{x-2}{x-2}\right) \quad \{\text{LCD}=(x-2)\} \\ &= \frac{(3x+1)-2(x-2)}{x-2} \\ &= \frac{3x+1-2x+4}{x-2} \\ &= \frac{x+5}{x-2} \end{aligned}$$

2 Write as a single fraction:

a $1+\frac{3}{x+2}$ **b** $-2+\frac{3}{x-4}$ **c** $-3-\frac{2}{x-1}$

d $\frac{2x-1}{x+1}+3$ **e** $3-\frac{x}{x+1}$ **f** $-1+\frac{4}{1-x}$

3 Write as a single fraction:

a $\frac{3}{x+2}+\frac{2}{x}$ **b** $\frac{1}{x-2}-\frac{1}{x-3}$ **c** $\frac{3}{x+1}-\frac{4}{x-2}$

d $\frac{5x}{x-4}+\frac{3x-2}{x+4}$ **e** $\frac{3x}{2x-5}+\frac{2x+5}{x-2}$ **f** $\frac{2x+1}{x-3}-\frac{x+4}{2x+1}$

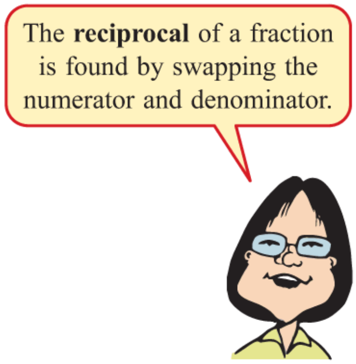
E MULTIPLYING AND DIVIDING ALGEBRAIC FRACTIONS

To **multiply** two or more fractions, we multiply the numerators to form the new numerator, and we multiply the denominators to form the new denominator.

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}=\frac{ac}{bd}$$

To **divide** by a fraction we multiply by its *reciprocal*.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$$

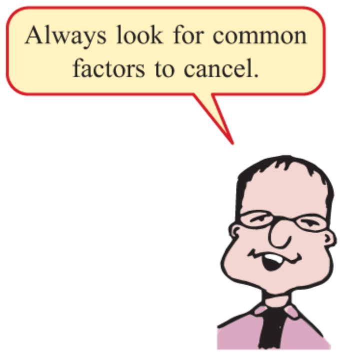


Example 9 Self Tutor

Find: **a** $\frac{x}{3}\times\frac{y}{x^2}$ **b** $\frac{2a}{b}\div\frac{6}{a}$

a
$$\frac{{}^{1}\cancel{x}}{3}\times\frac{y}{x^{\cancel{2}}}$$
$$=\frac{1\times y}{3\times x}$$
$$=\frac{y}{3x}$$

b
$$\frac{2a}{b}\div\frac{6}{a}$$
$$=\frac{2a}{b}\times\frac{a}{6}$$
$$=\frac{\cancel{2}a\times a}{b\times\cancel{6}\,3}$$
$$=\frac{a^2}{3b}$$



EXERCISE 2E

1 Find:

a $\frac{x}{4}\times\frac{7}{y}$ **b** $\frac{a}{5}\times\frac{5}{b}$ **c** $\frac{3}{x}\times\frac{y}{12}$

d $\frac{m^2}{4}\times\frac{6}{m}$ **e** $\frac{x}{6}\times\frac{2}{x}$ **f** $\frac{6}{t^2}\times 5$

g $\frac{x}{y^2}\times\frac{3y}{8}$ **h** $\frac{x}{y}\times\frac{y}{x}$ **i** $\left(\frac{4}{k}\right)^2$

j $\frac{10a^2}{b}\times\frac{3b}{2a}$ **k** $\frac{7m^3}{3}\times\frac{4}{mn}$ **l** $\frac{2}{x^2}\times\frac{5y}{x}\times\frac{x^5}{8y^2}$

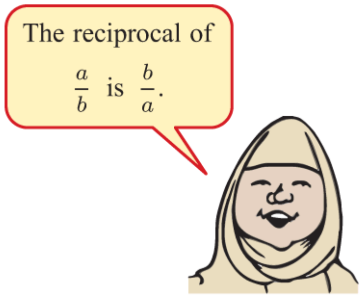
2 Find:

a $\frac{x}{3} \div \frac{2}{x}$ **b** $\frac{1}{x} \div \frac{x^2}{4}$ **c** $a \div \frac{a}{b}$

d $\frac{n}{5} \div \frac{n}{6}$ **e** $\frac{x^2}{2} \div 4$ **f** $\frac{a^2}{b} \div \frac{a}{2b}$

g $x^2 \div \frac{3}{x}$ **h** $\frac{2}{y} \div \frac{z}{3y}$ **i** $\frac{x}{2} \div \frac{xy}{6}$

j $\frac{a}{3b} \div \frac{5a}{b^2}$ **k** $\frac{8}{xy} \div 2x^2$ **l** $\frac{a^2}{6} \div \frac{a^3}{4b}$



F FACTORISATION

Algebraic **factorisation** is the reverse process of expansion.

For example, $2x^2 - 5x - 3$ can be *factorised* into two *linear factors*:

factorisation

$$2x^2 - 5x - 3 \quad = \quad (2x+1)(x-3)$$

expansion

When asked to factorise an expression:

Step 1: Look for any **common factors** to take out.

Step 2: Look for any of the following types:

- **difference between two squares** $a^2 - b^2 = (a+b)(a-b)$
- **perfect square** $a^2 + 2ab + b^2 = (a+b)^2$
- **sum and product type $x^2 + bx + c$** $x^2 + bx + c = (x+p)(x+q)$ where $p+q=b$ and $pq=c$.
- **sum and product type $ax^2 + bx + c$, $a \neq 0$ or 1**
  - Find two numbers p and q such that $p+q=b$ and $pq=ac$.
  - Replace bx by $px+qx$.
  - Complete the factorisation.

Example 10 **Self Tutor**

Fully factorise: **a** $3x^2 - 12x$ **b** $4x^2 - 1$ **c** $x^2 - 12x + 36$

a $3x^2 - 12x$ {$3x$ is a common factor}
$= 3x(x-4)$

b $4x^2 - 1$ {difference between two squares}
$= (2x)^2 - 1^2$
$= (2x+1)(2x-1)$

c $x^2 - 12x + 36$ {perfect square form}
$= x^2 + 2(x)(-6) + (-6)^2$
$= (x-6)^2$

EXERCISE 2F

1 Fully factorise:

- **a** $2x - 6$
- **b** $3x + 3$
- **c** $6 - 2x$
- **d** $x^2 + 2x$
- **e** $3x - x^2$
- **f** $3x^2 + 9x$
- **g** $2x^2 + 7x$
- **h** $4x^2 - 10x$
- **i** $6x^2 - 15x$

2 Fully factorise:

- **a** $x^2 - 36$
- **b** $9x^2 - 25$
- **c** $16x^2 - 1$
- **d** $2x^2 - 8$
- **e** $3x^2 - 9$
- **f** $4x^2 - 20$

3 Fully factorise:

- **a** $x^2 - 8x + 16$
- **b** $x^2 - 10x + 25$
- **c** $2x^2 - 8x + 8$
- **d** $16x^2 + 40x + 25$
- **e** $9x^2 + 12x + 4$
- **f** $x^2 - 22x + 121$

Example 11 Self Tutor

Fully factorise:

a $3x^2 + 12x + 9$ **b** $-x^2 + 3x + 10$

a $3x^2 + 12x + 9$ {3 is a common factor}
$= 3(x^2 + 4x + 3)$ {sum $= 4$, product $= 3$}
$= 3(x + 1)(x + 3)$

b $-x^2 + 3x + 10$
$= -(x^2 - 3x - 10)$ {removing -1 as a common factor to make the coefficient of x^2 be 1}
$= -(x - 5)(x + 2)$ {sum $= -3$, product $= -10$}

4 Fully factorise:

- **a** $x^2 + 9x + 8$
- **b** $x^2 + 7x + 12$
- **c** $x^2 - 7x - 18$
- **d** $x^2 + 4x - 21$
- **e** $x^2 - 9x + 18$
- **f** $x^2 + x - 6$
- **g** $-x^2 + x + 2$
- **h** $3x^2 - 42x + 99$
- **i** $-2x^2 - 4x - 2$
- **j** $2x^2 + 6x - 20$
- **k** $2x^2 - 10x - 48$
- **l** $-2x^2 + 14x - 12$
- **m** $-3x^2 + 6x - 3$
- **n** $-x^2 - 2x - 1$
- **o** $-5x^2 + 10x + 40$

Example 12 Self Tutor

Fully factorise:

a $2x^2 - x - 10$ **b** $6x^2 - 25x + 14$

a $2x^2 - x - 10$ has $ac = 2 \times -10 = -20$.
The factors of -20 which add to -1 are -5 and $+4$.

$$\therefore\ 2x^2 - x - 10 = 2x^2 - 5x + 4x - 10$$
$$= x(2x - 5) + 2(2x - 5)$$
$$= (2x - 5)(x + 2)$$

b $6x^2 - 25x + 14$ has $ac = 6 \times 14 = 84$.
The factors of 84 which add to -25 are -21 and -4.

$$\begin{aligned} \therefore \ 6x^2 - 25x + 14 &= 6x^2 - 21x - 4x + 14 \\ &= 3x(2x-7) - 2(2x-7) \\ &= (2x-7)(3x-2) \end{aligned}$$

5 Fully factorise:

a $2x^2 + 5x - 12$ **b** $3x^2 - 5x - 2$ **c** $7x^2 - 9x + 2$
d $6x^2 - x - 2$ **e** $4x^2 - 4x - 3$ **f** $10x^2 - x - 3$
g $2x^2 - 11x - 6$ **h** $3x^2 - 5x - 28$ **i** $8x^2 + 2x - 3$
j $10x^2 - 9x - 9$ **k** $3x^2 + 23x - 8$ **l** $6x^2 + 7x + 2$
m $-4x^2 - 2x + 6$ **n** $12x^2 - 16x - 3$ **o** $-6x^2 - 9x + 42$
p $21x - 10 - 9x^2$ **q** $8x^2 - 6x - 27$ **r** $12x^2 + 13x + 3$
s $12x^2 + 20x + 3$ **t** $15x^2 - 22x + 8$ **u** $14x^2 - 11x - 15$

G SUBSTITUTION

Given an algebraic expression, we can **substitute** values for the unknowns and hence **evaluate** the expression.

When we substitute a negative number, we place it in brackets to make sure the negative sign is evaluated correctly.

Example 13 Self Tutor

If $p = -2$, $q = 3$, and $r = 4$, find the value of:

a $p + 5q$ **b** $pr - 7q$ **c** $\dfrac{2r - 4q}{1 - pr}$

a
$$\begin{aligned} &p + 5q \\ &= (-2) + 5 \times 3 \\ &= -2 + 15 \\ &= 13 \end{aligned}$$

b
$$\begin{aligned} &pr - 7q \\ &= (-2) \times 4 - 7 \times 3 \\ &= -8 - 21 \\ &= -29 \end{aligned}$$

c
$$\begin{aligned} &\frac{2r - 4q}{1 - pr} \\ &= \frac{2 \times 4 - 4 \times 3}{1 - (-2) \times 4} \\ &= \frac{8 - 12}{1 - (-8)} \\ &= \frac{-4}{9} \end{aligned}$$

EXERCISE 2G

1 If $x = 5$ and $y = -1$, find the value of:

a $3x$ **b** $x + 2y$ **c** $-4y - x$ **d** $2xy$
e $-x + 3y$ **f** $6 - xy$ **g** $2x - 4y$ **h** $4xy + 2x$

2 If $l = 2$, $m = -3$, and $n = -1$, find the value of:

a $4l$ **b** $-n$ **c** $2mn$ **d** lmn
e $2l + m$ **f** $4m - 3l$ **g** $ml - 2n$ **h** $nl - 2mn$

3 If $x = 3$, $y = 2$, and $z = -1$, find the value of:

a $\dfrac{x}{y}$ **b** $\dfrac{x-y}{z}$ **c** $\dfrac{x+z}{2y}$ **d** $\dfrac{3z}{x+y}$

4 If $e = 4$, $f = 2$, and $g = -3$, evaluate:

a $\dfrac{g}{e}$ **b** $\dfrac{2g+e}{f}$ **c** $\dfrac{fg}{e}$ **d** $\dfrac{g-f}{e+g}$

Example 14 **Self Tutor**

If $x = 2$, $y = -4$, and $z = -5$, evaluate:

a y^2 **b** $yz^3 - 3x$

a y^2
$= (-4)^2$
$= 16$

b $yz^3 - 3x$
$= (-4) \times (-5)^3 - 3 \times 2$
$= 494$

5 If $a = 4$, $b = -1$, and $c = -3$, evaluate:

a b^2 **b** c^3 **c** $a^2 + c^2$ **d** $(a+c)^2$

e $a^3 + b^3$ **f** $(a+b)^3$ **g** $(2c)^2$ **h** $2c^2$

Example 15 **Self Tutor**

If $k = 5$, $l = -1$, and $m = 2$, evaluate:

a $\sqrt{k+l}$ **b** $\sqrt{m^2 + 3k}$

a $\sqrt{k+l}$
$= \sqrt{5 + (-1)}$
$= \sqrt{4}$
$= 2$

b $\sqrt{m^2 + 3k}$
$= \sqrt{2^2 + 3(5)}$
$= \sqrt{19}$

6 If $k = -2$, $l = 3$, and $m = 7$, evaluate:

a $\sqrt{l+k}$ **b** $\sqrt{m+3l}$ **c** $\sqrt{lm - 2k}$

d $\sqrt{k^2 + m^2}$ **e** $\sqrt{l^2 - m}$ **f** $\sqrt{m^2 + l^2}$

H LINEAR EQUATIONS

Linear equations are equations which can be written in the form $ax + b = 0$ where x is the **variable** or **unknown**, and a, b are constants.

To solve linear equations we need to rearrange the equation to **isolate** the unknown. We do this by looking at how the expression involving the unknown was built up, then undoing it using **inverse operations**. The inverse operations are performed on both sides of the equation to **maintain the balance**.

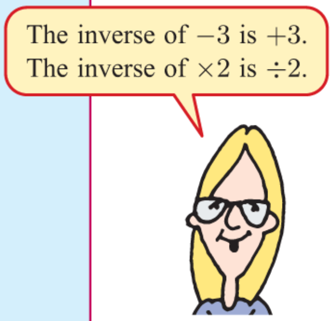
Once you have found a solution, you should check it is correct by **substituting** it back into the original equation.

Example 16 **Self Tutor**

Solve for x: **a** $2x - 3 = 5$ **b** $8 - 4x = -2$

a
$$\begin{aligned} 2x - 3 &= 5 \\ 2x - 3 + 3 &= 5 + 3 && \{\text{adding 3 to both sides}\} \\ \therefore\ 2x &= 8 \\ \therefore\ \frac{2x}{2} &= \frac{8}{2} && \{\text{dividing both sides by 2}\} \\ \therefore\ x &= 4 && \textit{Check: } 2 \times 4 - 3 = 8 - 3 = 5\ \checkmark \end{aligned}$$

b
$$\begin{aligned} 8 - 4x &= -2 \\ \therefore\ 8 - 4x - 8 &= -2 - 8 && \{\text{subtracting 8 from both sides}\} \\ \therefore\ -4x &= -10 \\ \therefore\ \frac{-4x}{-4} &= \frac{-10}{-4} && \{\text{dividing both sides by } -4\} \\ \therefore\ x &= \tfrac{5}{2} && \textit{Check: } 8 - 4 \times \left(\tfrac{5}{2}\right) = 8 - 10 = -2\ \checkmark \end{aligned}$$

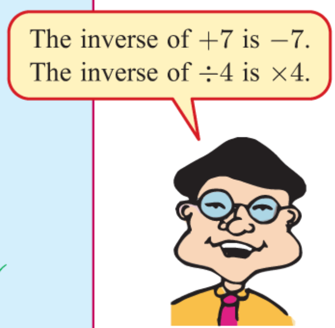


Example 17 **Self Tutor**

Solve for x: **a** $\frac{x}{4} + 7 = 5$ **b** $\frac{1}{3}(x + 2) = 6$

a
$$\begin{aligned} \frac{x}{4} + 7 &= 5 \\ \therefore\ \frac{x}{4} + 7 - 7 &= 5 - 7 && \{\text{subtracting 7 from both sides}\} \\ \therefore\ \frac{x}{4} &= -2 \\ \therefore\ \frac{x}{4} \times 4 &= -2 \times 4 && \{\text{multiplying both sides by 4}\} \\ \therefore\ x &= -8 && \textit{Check: } \tfrac{-8}{4} + 7 = -2 + 7 = 5\ \checkmark \end{aligned}$$

b
$$\begin{aligned} \tfrac{1}{3}(x + 2) &= 6 \\ \therefore\ \tfrac{1}{3}(x + 2) \times 3 &= 6 \times 3 && \{\text{multiplying both sides by 3}\} \\ \therefore\ x + 2 &= 18 \\ \therefore\ x + 2 - 2 &= 18 - 2 && \{\text{subtracting 2 from both sides}\} \\ \therefore\ x &= 16 && \textit{Check: } \tfrac{1}{3}(16 + 2) = \tfrac{1}{3} \times 18 = 6\ \checkmark \end{aligned}$$



EXERCISE 2H

1 Solve for x:

a $x + 5 = 3$ **b** $4x = 28$ **c** $-18 = -3x$ **d** $7 - x = 11$

e $2x + 3 = 14$ **f** $3x - 4 = -13$ **g** $5 - 2x = -9$ **h** $7 = 11 - 3x$

2 Solve for x:

a $\frac{x}{3} = 15$ **b** $\frac{1}{4}x = 16$ **c** $1 = \frac{x}{-3}$ **d** $\frac{x}{-5} = -7$

3 Solve for x:

a $\dfrac{x}{3} - 7 = 10$ **b** $\dfrac{x-4}{3} = -1$ **c** $\frac{1}{2}(x+5) = 6$

d $\dfrac{2x-3}{5} = 4$ **e** $\frac{1}{3}(2-x) = -5$ **f** $\dfrac{2-3x}{5} = -8$

Example 18 **Self Tutor**

Solve for x: $4(2x+5) - 3(x-2) = 16$

$$
\begin{aligned}
4(2x+5) - 3(x-2) &= 16 \\
\therefore\ 8x + 20 - 3x + 6 &= 16 && \{\text{expanding the brackets}\} \\
\therefore\ 5x + 26 &= 16 && \{\text{collecting like terms}\} \\
\therefore\ 5x + 26 - 26 &= 16 - 26 && \{\text{subtracting 26 from both sides}\} \\
\therefore\ 5x &= -10 \\
\therefore\ x &= -2 && \{\text{dividing both sides by 5}\}
\end{aligned}
$$

Check: $4(2 \times (-2) + 5) - 3((-2) - 2) = 4 \times 1 - 3 \times (-4)$
$= 4 + 12 = 16$ ✓

If the unknown appears more than once, look to expand brackets and collect like terms.

4 Solve for x:

a $3(x+2) + 2(x+4) = 19$ **b** $2(x-7) - 5(x+1) = -7$

c $3(x-3) - 4(x-5) = 2$ **d** $5(2x+1) - 3(x-1) = -6$

e $2(3x-2) + 7(2x+1) = 13$ **f** $4(x+4) + 3(5-2x) = 19$

Example 19 **Self Tutor**

Solve for x: $5 - 3(-1+x) = x$

$$
\begin{aligned}
5 - 3(-1+x) &= x \\
\therefore\ 5 + 3 - 3x &= x && \{\text{expanding the brackets}\} \\
\therefore\ 8 - 3x &= x \\
\therefore\ 8 - 3x + 3x &= x + 3x && \{\text{adding } 3x \text{ to both sides}\} \\
\therefore\ 8 &= 4x \\
\therefore\ \frac{8}{4} &= \frac{4x}{4} && \{\text{dividing both sides by 4}\} \\
\therefore\ 2 &= x \quad \text{or} \quad x = 2
\end{aligned}
$$

Check: LHS $= 5 - 3(-1+2) = 5 - 3 \times 1 = 2 =$ RHS ✓

If the unknown appears on both sides, use inverse operations to remove it from one side.

5 Solve for x:

a $5x - 5 = 4x + 1$ **b** $2x - 3 = 6 - x$ **c** $1 - 3x = 2x - 9$

d $-4x = 8 - 2x$ **e** $9 - 5x = x + 6$ **f** $4x - 7 = x + 3$

6 Solve for x:

a $6 - x + 3(1 - x) = 7 - 2x$
b $8 - 5(3 - x) = 9 + x$
c $6 + 7x - 2(3 - x) = 5x - 8$
d $5(3x + 1) - 4x = x - 2$

7 Explain why the equation $5(2x - 1) + 2 = 10x - 3$ is true for all values of x.

8 Show that the equation $2(9x - 1) = 6(3x + 1)$ has no solutions.

Example 20 **Self Tutor**

Solve for x: $\dfrac{2-x}{3} = \dfrac{x}{5}$

$$\frac{2-x}{3} = \frac{x}{5} \qquad \{\text{LCD of fractions} = 15\}$$

$$\therefore \left(\frac{2-x}{3}\right) \times \frac{5}{5} = \frac{x}{5} \times \frac{3}{3} \qquad \{\text{to create the common denominator}\}$$

$$\therefore \frac{5(2-x)}{15} = \frac{3x}{15}$$

$$\therefore 5(2-x) = 3x \qquad \{\text{equating numerators}\}$$

$$\therefore 10 - 5x = 3x \qquad \{\text{expanding the brackets}\}$$

$$\therefore 10 = 8x \qquad \{\text{adding } 5x \text{ to both sides}\}$$

$$\therefore \tfrac{5}{4} = x \qquad \{\text{dividing both sides by } 8\}$$

For equations involving fractions, write the fractions with the lowest common denominator (LCD) then equate numerators.

9 Solve for x:

a $\dfrac{x}{5} = \dfrac{2}{3}$
b $\dfrac{5}{6} = \dfrac{x}{5}$
c $\dfrac{x-3}{2} = \dfrac{1}{5}$
d $\dfrac{x}{4} = \dfrac{1+x}{3}$
e $\dfrac{2-x}{5} = \dfrac{x}{3}$
f $\dfrac{x-1}{5} = -\dfrac{x}{7}$

I LINEAR INEQUALITIES

Linear inequalities take the same form as linear equations, except they contain an inequality sign $>$, $<$, $\geqslant$, or $\leqslant$ instead of an equality sign $=$.

$3x \leqslant 27$ and $1 - 4x > 8$ are examples of linear inequalities.

When solving **linear inequalities**, we perform the same operations as when we solve linear equations except:

- **Interchanging** the sides **reverses** the inequality sign.
- **Multiplying** or **dividing** both sides by a **negative reverses** the inequality sign.
- We do not multiply or divide both sides by an expression involving the unknown. This is because we do not know its sign.

Example 21 **Self Tutor**

Solve for x:

a $4x - 5 > 15$

b $7 - 2x \leqslant 20$

a $4x - 5 > 15$

$\therefore\ 4x - 5 + 5 > 15 + 5$ {adding 5 to both sides}

$\therefore\ 4x > 20$

$\therefore\ \frac{4x}{4} > \frac{20}{4}$ {dividing both sides by 4}

$\therefore\ x > 5$

b $7 - 2x \leqslant 20$

$\therefore\ 7 - 2x - 7 \leqslant 20 - 7$ {subtracting 7 from both sides}

$\therefore\ -2x \leqslant 13$

$\therefore\ \frac{-2x}{-2} \geqslant \frac{13}{-2}$ {dividing both sides by -2, we reverse the inequality sign}

$\therefore\ x \geqslant -\frac{13}{2}$

EXERCISE 2I

1 Solve for x:

a $x - 6 > 3$
b $3x \leqslant 21$
c $4 + x \geqslant 2$
d $\frac{x}{4} < -1$
e $2x + 1 \leqslant 19$
f $7 - x > 16$
g $3x - 4 \geqslant 11$
h $\frac{x}{3} + 5 < 4$
i $7 - \frac{x}{2} > 4$

2 Solve for x:

a $\frac{x+3}{2} \leqslant 5$
b $\frac{9-x}{5} < 3$
c $\frac{2x-4}{3} \geqslant -1$

3 Solve for x:

a $3 + 2x > 1 - x$
b $6x + 13 \leqslant 3x + 1$
c $\frac{x}{2} - 4 > x + 7$
d $5(x-1) + 2 \geqslant 6$
e $4(x-3) < 3(x+5)$
f $6(5-x) \leqslant x - 3(x-2)$

J RATIONAL EQUATIONS

A **rational equation** is an equation in which the unknown appears in the denominator of a fraction.

Many simple rational equations can be converted into linear equations by writing all fractions with the lowest common denominator, and then equating numerators.

Example 22 **Self Tutor**

Solve for x: $\dfrac{7}{x+1} = \dfrac{3}{x}$

$$\frac{7}{x+1} = \frac{3}{x} \qquad \{\text{LCD of fractions} = x(x+1)\}$$

$$\therefore \left(\frac{7}{x+1}\right) \times \frac{x}{x} = \frac{3}{x} \times \left(\frac{x+1}{x+1}\right) \qquad \{\text{to create the common denominator}\}$$

$$\therefore \frac{7x}{x(x+1)} = \frac{3(x+1)}{x(x+1)}$$

$$\therefore 7x = 3(x+1) \qquad \{\text{equating numerators}\}$$

$$\therefore 7x = 3x + 3 \qquad \{\text{expanding the brackets}\}$$

$$\therefore 4x = 3 \qquad \{\text{subtracting } 3x \text{ from both sides}\}$$

$$\therefore x = \tfrac{3}{4} \qquad \{\text{dividing both sides by } 4\}$$

EXERCISE 2J

1 Solve for x:

a $\dfrac{3}{x} = \dfrac{2}{5}$ **b** $\dfrac{2}{x} = \dfrac{1}{3}$ **c** $\dfrac{7}{x} = \dfrac{3}{4}$

d $\dfrac{5}{2x} = \dfrac{4}{3}$ **e** $\dfrac{4}{x+1} = \dfrac{5}{3}$ **f** $\dfrac{3}{1-x} = \dfrac{2}{5}$

2 Explain why $\dfrac{4}{3x} = \dfrac{5}{4x}$ has no solutions.

3 Solve for x:

a $\dfrac{1}{x} = \dfrac{2}{x+1}$ **b** $\dfrac{x}{x+4} = \dfrac{1}{2}$ **c** $\dfrac{5}{x} = \dfrac{8}{x-2}$

d $\dfrac{x}{2x-1} = -\dfrac{1}{3}$ **e** $\dfrac{2}{5} = \dfrac{x-3}{3x}$ **f** $\dfrac{4}{x+3} = \dfrac{7}{2x}$

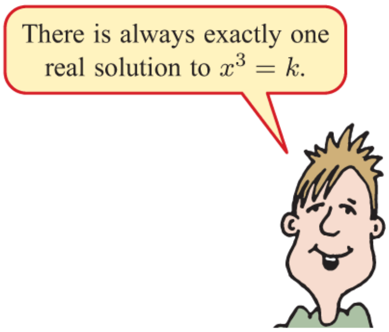
K POWER EQUATIONS

A **power equation** is an equation of the form $x^n = k$ where $n \neq 0$.

In this Section we consider the cases $n = 2$ and $n = 3$.

If $x^2 = k$ then $\begin{cases} x = \pm\sqrt{k} & \text{if } k > 0 \\ x = 0 & \text{if } k = 0 \\ x \text{ has no real solutions if } k < 0. \end{cases}$

If $x^3 = k$ then $x = \sqrt[3]{k}$.



Example 23 **Self Tutor**

Solve for x:

a $x^2 = 4$ **b** $x^2 = -5$ **c** $x^3 = 45$

a $x^2 = 4$
$\therefore\ x = \pm\sqrt{4}$
$\therefore\ x = \pm 2$

b $x^2 = -5$ has no real solutions.

c $x^3 = 45$
$\therefore\ x = \sqrt[3]{45}$
$\therefore\ x \approx 3.56$

EXERCISE 2K

1 Solve for x:

a $x^2 = 9$ **b** $x^2 = 49$ **c** $x^2 = 36$ **d** $x^2 = 0$
e $x^2 = 1$ **f** $x^2 = 17$ **g** $x^2 = 23$ **h** $x^2 = 100$
i $x^2 = -4$ **j** $x^2 = -7$ **k** $x^2 = 27$ **l** $x^2 = -27$

2 Solve for x:

a $x^3 = 8$ **b** $x^3 = 27$ **c** $x^3 = 0$ **d** $x^3 = 64$
e $x^3 = -1$ **f** $x^3 = -125$ **g** $x^3 = \frac{1}{8}$ **h** $x^3 = 36$

Example 24 **Self Tutor**

Solve for x:

a $x^2 + 4 = 9$ **b** $x^3 + 27 = 0$

a $x^2 + 4 = 9$
$\therefore\ x^2 = 5$ {subtracting 4 from both sides}
$\therefore\ x = \pm\sqrt{5}$

b $x^3 + 27 = 0$
$\therefore\ x^3 = -27$ {subtracting 27 from both sides}
$\therefore\ x = \sqrt[3]{-27}$
$\therefore\ x = -3$

3 Solve for x:

a $x^2 + 5 = 9$ **b** $x^2 + 16 = 25$ **c** $x^2 + 2 = 27$
d $x^2 + 7 = 23$ **e** $8 + x^2 = 44$ **f** $x^2 + 14 = 39$
g $10 + x^2 = 60$ **h** $x^2 + 5 = 49$ **i** $6 + x^2 = 18$

4 Solve for x:

a $x^3 - 1 = 0$ **b** $x^3 + 8 = 0$ **c** $x^3 - 216 = 0$
d $x^3 - 1\,000\,000 = 0$ **e** $x^3 - 16 = 0$ **f** $x^3 + 800 = 0$

L FORMULAE

A **formula** is an equation which connects two or more variables.
The plural of formula is **formulae** or **formulas**.

It is usual to write a formula so that one of the variables is on its own on one side of the equation, and the other variable(s) are contained in an expression on the other side.

The variable on its own is called the **subject** of the formula.

If the formula contains two or more variables and we know the value of all but one of them, we can solve an equation to find the remaining variable.

Step 1: Write down the formula and state the values of the known variables.
Step 2: Substitute the known values into the formula to form a one variable equation.
Step 3: Solve the equation for the unknown variable.

Example 25 Self Tutor

The acceleration of a falling raindrop is given by $a = g - 1.96v$ m s^{-2}, where $g = 9.8$ m s^{-2} is the gravitational constant, and v is the speed of the raindrop in m s^{-1}. Find:

a the acceleration of the raindrop before it starts falling
b the acceleration of the raindrop when its speed reaches 3 m s^{-1}
c the speed of the raindrop for which it does not accelerate.

a $a = g - 1.96v$ where $g = 9.8$ and $v = 0$
$\therefore\ a = 9.8 - 1.96 \times 0$
$\therefore\ a = 9.8$ m s^{-2}

b $a = g - 1.96v$ where $g = 9.8$ and $v = 3$
$\therefore\ a = 9.8 - 1.96 \times 3$
$\therefore\ a = 3.92$ m s^{-2}

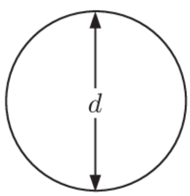
c $a = g - 1.96v$ where $a = 0$ and $g = 9.8$
$\therefore\ 0 = 9.8 - 1.96v$
$\therefore\ 1.96v = 9.8$
$\therefore\ v = \dfrac{9.8}{1.96} = 5$ m s^{-1}

EXERCISE 2L

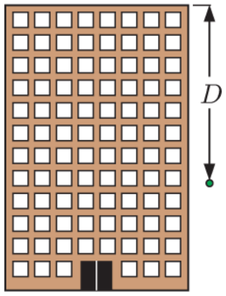
1 For the formula $P = 2m + 3n$, find the value of P when:

a $m = 4$ and $n = 5$
b $m = 6$ and $n = -1$
c $m = -3$ and $n = 2$
d $m = -2.5$ and $n = -7$.

2 The circumference C of a circle with diameter d is given by the formula $C = \pi d$ where $\pi \approx 3.141\,59$ is a constant. Find the circumference of a circle with diameter:



a 8 cm
b 11 m
c 15.7 cm.

3 

A tennis ball is dropped from the top of a tall building. The total distance it has fallen is given by the formula $D = \frac{1}{2}gt^2$ where D is the distance in metres and t is the time taken in seconds. $g = 9.8 \text{ m s}^{-2}$ is the gravitational constant.

a Find the total distance fallen in the first 3 seconds.

b Find the height of the building, to the nearest metre, if the ball takes 5.13 seconds to reach the ground.

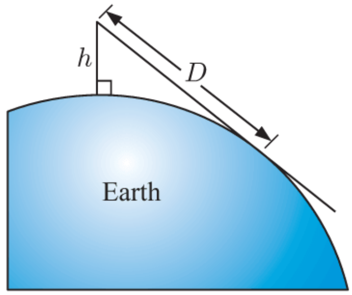
4 The formula $D = 3.56\sqrt{h}$ estimates the distance in kilometres to the horizon which is seen by a person with eye level h metres above the level of the sea.

Find the distance to the horizon when a person's eye level is:

a 2 m above sea level

b 10 m above sea level

c 20 m above sea level.



5 When a car travels d kilometres in time t hours, the average speed for the journey is given by $s = \frac{d}{t} \text{ km h}^{-1}$. Find:

a the average speed of a car which travels 200 km in $2\frac{1}{2}$ hours

b the distance travelled by a car in $3\frac{1}{4}$ hours if its average speed is 80 km h^{-1}

c the time taken, to the nearest minute, to travel 865 km at an average speed of 110 km h^{-1}.

6 The potential difference V across an R ohm resistor is given by $V = IR$ volts, where I is the current in amps flowing through the circuit. Find:

a the potential difference across a 6 ohm resistor if the current in the circuit is 0.08 amps

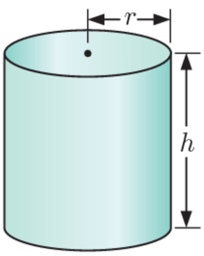
b the resistance in a circuit with current 0.2 amps if the potential difference is 12 volts.

7 The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$. Find:

a the volume of a cylindrical tin can of radius 12 cm and height 17.5 cm

b the height of a cylinder of radius 4 cm if its volume is 80 cm^3

c the radius, in mm, of copper wire with volume 100 cm^3 and length 90 cm.



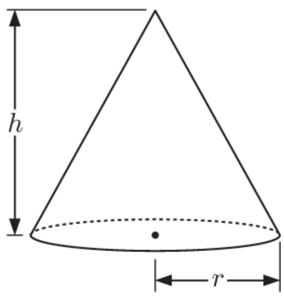
M FORMULA REARRANGEMENT

Consider the formula $V = \frac{1}{3}\pi r^2 h$, which gives the volume of a cone with radius r and height h.

We say that V is the **subject** of the formula because V is expressed in terms of the other variables r and h.

The formula can be **rearranged** to make an **equivalent** formula in which one of the other variables is the subject.

For example, the height of the cone is given by $h = \frac{3V}{\pi r^2}$.



We rearrange formulae using the same methods which we used to solve equations. We perform **inverse operations** to isolate the variable we wish to make the subject.

Example 26 Self Tutor

Make y the subject of $3x - 7y = 22$.

$$3x - 7y = 22$$
$$\therefore\ -7y = 22 - 3x \quad \{\text{subtracting } 3x \text{ from both sides}\}$$
$$\therefore\ 7y = 3x - 22 \quad \{\text{multiplying both sides by } -1\}$$
$$\therefore\ y = \frac{3x-22}{7} \quad \{\text{dividing both sides by } 7\}$$

EXERCISE 2M

1 Make y the subject of:

a $x + 2y = 4$ **b** $2x + 6y = 7$ **c** $3x + 4y = 11$
d $5x + 4y = 8$ **e** $7x + 2y = 20$ **f** $11x + 15y = 38$

2 Make y the subject of:

a $x - 2y = 4$ **b** $2x - 6y = 7$ **c** $3x - 4y = -12$
d $4x - 5y = 18$ **e** $7x - 6y = 42$ **f** $12x - 13y = -44$

3 Make x the subject of:

a $a + x = b$ **b** $ax = b$ **c** $2x + a = d$
d $c + x = t$ **e** $7x + 3y = d$ **f** $ax + by = c$
g $mx - y = c$ **h** $c - 2x = p$ **i** $a - 3x = t$
j $n - kx = 5$ **k** $a - bx = n$ **l** $p = a - nx$

Example 27 Self Tutor

Make z the subject of $y = \frac{x}{z}$.

$$y = \frac{x}{z}$$
$$\therefore\ yz = x \quad \{\text{multiplying both sides by } z\}$$
$$\therefore\ z = \frac{x}{y} \quad \{\text{dividing both sides by } y\}$$

4 Make x the subject of:

a $a = \frac{x}{b}$ **b** $\frac{a}{x} = d$ **c** $p = \frac{2}{x}$
d $\frac{x}{2} = n$ **e** $\frac{5}{x} = \frac{y}{z}$ **f** $\frac{m}{x} = \frac{x}{n}$

5 The equation of a straight line is $5x + 3y = 18$. Rearrange this formula into the form $y = mx + c$, and hence state the gradient m and the y-intercept c.

Example 28 Self Tutor

The circumference of a circle is given by $C = 2\pi r$, where r is the circle's radius.

a Rearrange this formula to make r the subject.

b Hence find the radius when the circumference is:

i 10 cm **ii** 20 cm **iii** 50 cm.

a $2\pi r = C$

$\therefore\ r = \dfrac{C}{2\pi}$ {dividing both sides by 2π}

b **i** When $C = 10$, $r = \dfrac{10}{2\pi} \approx 1.59$

$\therefore$ the radius is about 1.59 cm.

ii When $C = 20$, $r = \dfrac{20}{2\pi} \approx 3.18$

$\therefore$ the radius is about 3.18 cm.

iii When $C = 50$, $r = \dfrac{50}{2\pi} \approx 7.96$

$\therefore$ the radius is about 7.96 cm.

6 **a** Make s the subject of the formula $R = 5s + 2t$.

b Find the value of s when:

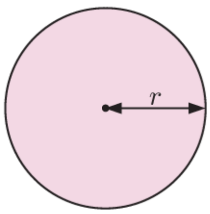
i $R = 16$ and $t = 3$ **ii** $R = 2$ and $t = 11$ **iii** $R = 8$ and $t = -2$.

7 **a** Make x the subject of the formula $M = \dfrac{t}{2xy}$.

b Find the value of x when:

i $M = 5$, $t = 4$, $y = 6$ **ii** $M = 20$, $t = 5$, $y = 0.1$

8 The area of a circle of radius r is given by $A = \pi r^2$.



a Make r the subject of the formula.

b Find the radius of a circle with area:

i 4 cm^2 **ii** 12.5 cm^2 **iii** 25 cm^2.

9 When a car travels d kilometres in time t hours, the average speed s for the journey is given by the formula $s = \dfrac{d}{t}$ km h^{-1}.

a Make d the subject of the formula. Hence find the distance travelled by a car if:

i the average speed is 60 km h^{-1} and the time travelled is 3 hours

ii the average speed is 80 km h^{-1} and the time travelled is $1\frac{1}{2}$ hours

iii the average speed is 95 km h^{-1} and the time travelled is 1 h 20 min.

b Make t the subject of the formula. Hence find the time required for a car to travel:

i 180 km at an average speed of 60 km h^{-1}

ii 140 km at an average speed of 35 km h^{-1}

iii 220 km at an average speed of 100 km h^{-1}.

Background Knowledge 3

Measurement

Contents:

A INTERNATIONAL SYSTEM (SI) UNITS

HISTORICAL NOTE

The decimal **Metric** system of measurement was created at the time of the French Revolution.

Having decided that a new unit of length, the metre, should be one ten millionth of the distance from the North Pole to the Equator, **Pierre Méchain** (1744 - 1804) and **Jean-Baptiste Delambre** (1749 - 1822) set about surveying the 1000 km section of the meridian arc from Dunkirk to Barcelona. At the end of their survey, a platinum bar was deposited in the Archives de la République in Paris in 1799, defining the standard metre.

Pierre Méchain

Jean-Baptiste Delambre

Despite errors in the calculations of Méchain and Jean-Baptiste, when it was decided in 1867 to create a new international standard metre, the length was taken to be that of the platinum bar still in Paris.

The **International System of Units**, abbreviated SI from the French *le Système International d'Unités*, is the world's most widely used system of measurement.

It is founded on seven base units:

Quantity	*Name*	*Symbol*
Distance	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Intensity of light	candela	cd
Amount of substance	mole	mol

Other SI units, called **derived units**, are defined in terms of the base units by multiplying or dividing them. The result is often given its own special name.

Some of the common SI derived units are:

Quantity	*Name*	*Symbol*
Area	square metre	m^2
Volume	cubic metre	m^3
Mass	gram	g
Velocity	metres per second	$m\,s^{-1}$
Angle	radian	rad

Quantity	*Name*	*Symbol*
Force	newton	N
Pressure	pascal	Pa
Energy	joule	J
Power	watt	W
Frequency	hertz	Hz

When we multiply one unit by another, we leave a short space between the unit symbols.

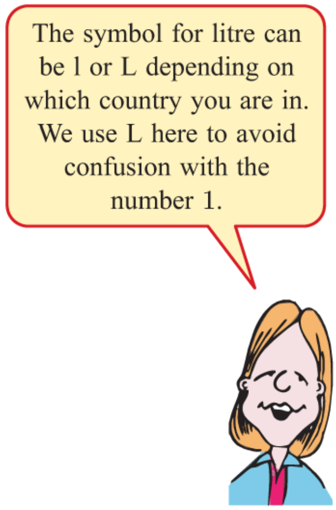
For example, *torque* or a *moment of force* is measured in newton metres, written N m.

When we divide one unit by another, we use an oblique line between the unit symbols, or a negative exponent.

For example:
- speed in metres per second is written m/s or m s^{-1}
- the *density* of an object is its mass per unit volume, measured in kg/m^3 or kg m^{-3}.

In addition to the base and derived units, the SI allows the use of other units, such as:

Quantity	*Name*	*Symbol*	*SI equivalent*
Time	minute hour	min h	60 s 3600 s
Mass	tonne	t	1000 kg
Capacity	litre	L	$0.001\ \text{m}^3$
Area	hectare	ha	$10\,000\ \text{m}^2$
Angle	degree	°	$\frac{\pi}{180}$ rad
Temperature	degree Celsius	°C	$K - 273.15$
Pressure	millibar	mb	100 Pa
Distance at sea	nautical mile	Nm	1.852 km
Speed at sea	knot	kn	$1.852\ \text{km h}^{-1}$
Energy	kilowatt hour	kWh	3.6 MJ



Smaller or larger multiples of SI units are obtained by combining the base unit with a prefix chosen according to powers of 10. The most commonly used are:

nano	n	$10^{-9} = \frac{1}{1\,000\,000\,000}$
micro	μ	$10^{-6} = \frac{1}{1\,000\,000}$
milli	m	$10^{-3} = \frac{1}{1000}$

kilo	k	$10^3 = 1000$
mega	M	$10^6 = 1\,000\,000$
giga	G	$10^9 = 1\,000\,000\,000$

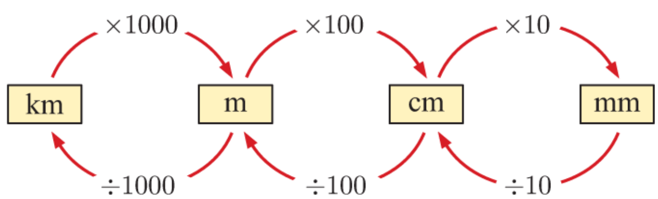
The SI also accepts the prefix "centi" (10^{-2}) which can be used in conjunction with metre, litre, or gram.

When stating the value of a measurement, the prefix chosen should give the value as a number between 0.1 and 1000. Thus, one nautical mile is written as 1.852 km, not 1852 m.

For more information on SI units, visit www.bipm.org/en/publications/si-brochure/

UNITS OF LENGTH

The most common units of length are:
- **millimetres** (mm)
- **centimetres** (cm)
- **metres** (m)
- **kilometres** (km)



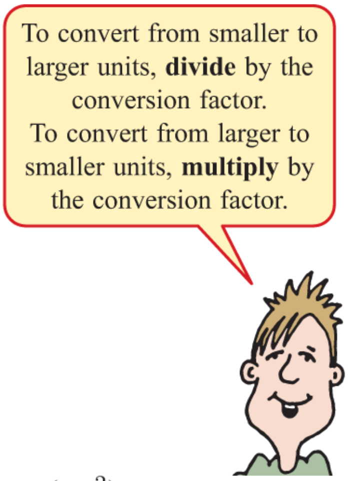
Example 1 **Self Tutor**

Convert:

a 16.73 m to cm **b** 48 380 cm to km

a 16.73 m
$= (16.73 \times 100) \text{ cm}$
$= 1673 \text{ cm}$

b $48\,380 \text{ cm}$
$= (48\,380 \div 100 \div 1000) \text{ km}$
$= 0.4838 \text{ km}$



UNITS OF AREA

The most common units of area are the squares of the length units:

- **square millimetres** (mm^2)
- **square centimetres** (cm^2)
- **square metres** (m^2)
- **square kilometres** (km^2).

For larger areas we can also use **hectares** (ha).

$$\begin{tikzcd}
\boxed{\text{km}^2} \arrow[r, bend left, "\times 100"] & \boxed{\text{ha}} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10\,000"] & \boxed{\text{m}^2} \arrow[l, bend left, "\div 10\,000"] \arrow[r, bend left, "\times 10\,000"] & \boxed{\text{cm}^2} \arrow[l, bend left, "\div 10\,000"] \arrow[r, bend left, "\times 100"] & \boxed{\text{mm}^2} \arrow[l, bend left, "\div 100"]
\end{tikzcd}$$

Example 2 **Self Tutor**

Convert:

a 650 000 m^2 to ha **b** 2.5 m^2 to mm^2

a $650\,000 \text{ m}^2$
$= (650\,000 \div 10\,000) \text{ ha}$
$= 65 \text{ ha}$

b 2.5 m^2
$= (2.5 \times 10\,000 \times 100) \text{ mm}^2$
$= 2\,500\,000 \text{ mm}^2$

UNITS OF VOLUME

The most common units of volume are the cubes of the length units:

- **cubic millimetres** (mm^3)
- **cubic centimetres** (cm^3)
- **cubic metres** (m^3)

$$\begin{tikzcd}
\boxed{\text{m}^3} \arrow[r, bend left, "\times 100^3"] & \boxed{\text{cm}^3} \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \boxed{\text{mm}^3} \arrow[l, bend left, "\div 10^3"]
\end{tikzcd}$$

UNITS OF CAPACITY

The most common units of capacity are:

- **millilitres** (mL)
- **litres** (L)
- **kilolitres** (kL)

$$\begin{tikzcd}
\boxed{\text{kL}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{L}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{mL}} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

Example 3 **Self Tutor**

Convert:

a 3.7 L to mL **b** 57 620 L to kL

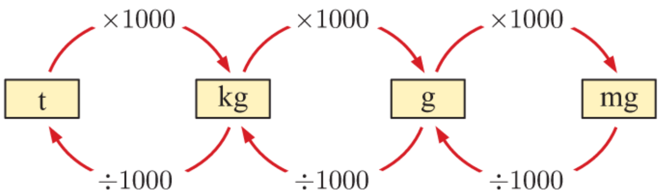
a $3.7 \text{ L} = (3.7 \times 1000) \text{ mL}$
$= 3700 \text{ mL}$

b $57\,620 \text{ L} = (57\,620 \div 1000) \text{ kL}$
$= 57.62 \text{ kL}$

UNITS OF MASS

The most common units of mass are:

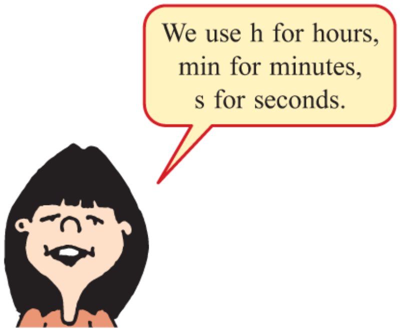
- **milligrams** (mg)
- **grams** (g)
- **kilograms** (kg)
- **tonnes** (t)



UNITS OF TIME

The most common units of time are related as follows:

1 **minute** = 60 seconds
1 **hour** = 60 minutes = 3600 seconds
1 **day** = 24 hours
1 **week** = 7 days
1 **year** = 12 months = $365\frac{1}{4}$ days



Example 4 **Self Tutor**

Convert:

a 2 hours 18 minutes 26 seconds to seconds **b** 7200 seconds to minutes

a 2 h 18 min 26 s
$= (2 \times 3600) \text{ s} + (18 \times 60) \text{ s} + 26 \text{ s}$
$= 8306 \text{ s}$

b 7200 s
$= (7200 \div 60) \text{ min}$
$= 120 \text{ min}$

EXERCISE 3A

1 Convert:

a 8250 cm to m **b** 295 mm to cm **c** 6250 m to km
d 73.8 mm to cm **e** 24.63 cm to m **f** 9.761 m to km

2 Convert:

a 413 cm to mm **b** 3754 km to m **c** 4.829 km to cm
d 26.9 m to mm **e** 0.47 km to cm **f** 3.88 km to mm

3 A hardware store stocks 85 reels of garden hose, each containing 30 m of hose. How many kilometres of garden hose are stocked by the store?

4 Phyllis is a candlemaker. The wick of each of her candles is 27.5 cm long. If Phyllis has 770 m of wick, how many candles can she make?

5 In 18th century Europe, the unit of length *ell* had multiple meanings. For example, the Polish ell was approximately 78.7 cm, and the French ell was approximately 137.2 cm.
A polish tailor asked his apprentice, who was travelling to France, to purchase 18 ells of silk. Unfortunately, the apprentice purchased the silk in French ells instead of Polish ells. How much extra silk was purchased? Give your answer to the nearest Polish ell.

6 Convert:

a 560 cm^2 to m^2 **b** 55 mm^2 to cm^2 **c** 8.43 m^2 to km^2
d 3820 m^2 to ha **e** 7.21 ha to km^2 **f** 9 890 000 mm^2 to m^2

7 Convert:

a 2.1 cm^2 to mm^2 **b** 0.38 m^2 to cm^2 **c** 4.8 km^2 to ha
d 0.0059 ha to m^2 **e** 83.25 ha to m^2 **f** 0.13 km^2 to m^2

8 In my city there are around 56 800 houses built on 4800 hectares of land. Find, in square metres, the average lot size for a house.

9 A glazing company manufactures glass in 9.9 m^2 sheets. It then cuts the sheets into 550 cm^2 panels. How many panels can be cut from each sheet?

10 Convert:

a 39 100 000 cm^3 to m^3 **b** 0.51 cm^3 to mm^3 **c** 469 000 cm^3 to m^3
d 3.82 m^3 to cm^3 **e** 5.27 mm^3 to cm^3 **f** 0.0179 m^3 to cm^3
g 692 000 mm^3 to cm^3 **h** 183 460 000 mm^3 to m^3 **i** 0.0051 m^3 to cm^3

11 A steel ball bearing has volume 0.27 cm^3. If the manufacturer has 1.35 m^3 of steel, how many ball bearings can be made?

12 There is about 5.5 cm^3 of aluminium in a soft drink can. A recycling depot collects 46 291 cans and melts them down, losing 15% of the metal in the process. How many cubic metres of aluminium do they now have?

13 Convert:

a 4.21 L to mL **b** 8.63 kL to L **c** 4600 mL to L
d 56 900 L to kL **e** 3970 mL to kL **f** 0.012 kL to mL

14 A petrol station begins the week with 143 kL of petrol in its storage tanks. During the week, 2856 people fill up their car tanks, using an average of 35.8 L each. How much petrol is left at the station?

15 A citrus oil producer sells oil in 200 mL bottles. How many bottles can be filled from a 372 L batch of citrus oil?

16 Convert:

- **a** 5.9 kg to g
- **b** 2600 g to kg
- **c** 3750 g to t
- **d** 15 kg to mg
- **e** 4.8 t to g
- **f** 1600 mg to kg
- **g** 1.7385 g to mg
- **h** 46 218 g to t
- **i** 0.0361 kg to mg

17 How many 6 gram nails can be made from 0.12 t of iron?

18 Dominic found that the average mass of a banana on his plantation was 138 grams. That year he exported 45 000 bananas.

- **a** How many tonnes of bananas did Dominic export?
- **b** If each truck carried 1700 kg of bananas to the port, how many truck loads were required that year?

19 In jewellery, the terms *carat* and *karat* are often confused. A *carat* is a unit of weight equal to 200 mg. A *karat* is a unit for the purity of gold, equivalent to $\frac{1}{24}$ of the whole.

- **a** Find the weight in grams of a 16 carat diamond.
- **b** A jeweller melts 21 g of 8 karat gold with 27 g of 24 karat gold. How many karat is the resulting blend?

20 Convert to seconds:

- **a** 15 minutes
- **b** 42 minutes
- **c** 1 hour 20 minutes
- **d** 3 min 47 s
- **e** 3 h 6 min 18 s
- **f** 5 h 16 min 25 s

21 Convert to minutes:

- **a** $4\frac{1}{2}$ hours
- **b** 2040 s
- **c** 5 h 23 min

Example 5 **Self Tutor**

Calculate in hours and minutes:

- **a** 3 h 42 min + 1 h 37 min
- **b** 4 h 25 min − 2 h 34 min

a
$$\begin{array}{rr} & 3\text{ h } 42\text{ min} \\ + & 1\text{ h } 37\text{ min} \\ \hline & 4\text{ h } 79\text{ min} \end{array}$$

Since 60 min = 1 h, the total is 5 h 19 min.

b
$$\begin{array}{rr} & \overset{3}{\cancel{4}}\text{ h } \overset{85}{\cancel{25}}\text{ min} \\ - & 2\text{ h } 34\text{ min} \\ \hline & 1\text{ h } 51\text{ min} \end{array}$$

22 Calculate in hours and minutes:

- **a** 1 h 14 min + 2 h 17 min
- **b** 3 h 41 min + 2 h 24 min
- **c** 9 h − 5 h 55 min
- **d** 7 h 23 min − 4 h 48 min

Example 6 — Self Tutor

Convert:

a 15 knots to kilometres per hour

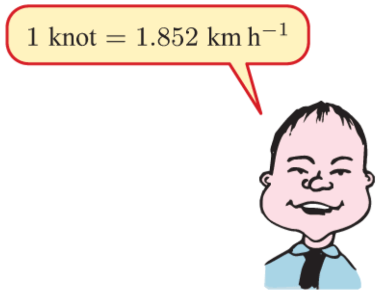
b 36 km h^{-1} to m s^{-1}

a
$$\begin{aligned} 1 \text{ kn} &= 1.852 \text{ km h}^{-1} \\ 15 \text{ kn} &= 15 \times 1.852 \\ &= 27.78 \text{ km h}^{-1} \end{aligned}$$

b
$1 \text{ km} = 1000 \text{ m}$
and $1 \text{ h} = 3600 \text{ s}$
$$\therefore \; 36 \text{ km h}^{-1} = \frac{(36 \times 1000) \text{ m}}{3600 \text{ s}} = 10 \text{ m s}^{-1}$$

23 Convert:

a 40 kn to km h^{-1}
b 20 km h^{-1} to kn
c 55 km h^{-1} to m s^{-1}
d 30 m s^{-1} to km h^{-1}
e 3.8 km h^{-1} to m s^{-1}
f 1.24 m s^{-1} to km h^{-1}



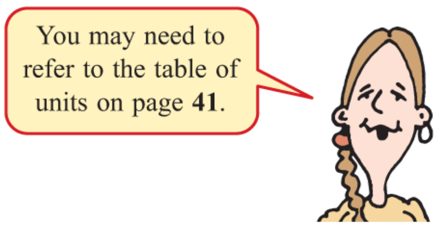
24 Two cars travelling on a country road have speeds 20.8 m s^{-1} and 71 km h^{-1}. Which speed is faster?

25 Convert:

a $0°$C to K
b 97.2 MJ to kWh
c 3.5 Nm to km

26 Convert:

a $26\,580$ ns to μs
b 3.2 μm to nm
c 2530 ML to GL
d 0.2 mg to μg
e 600 μL to mL
f 68.5 μg to mg



B PERIMETER

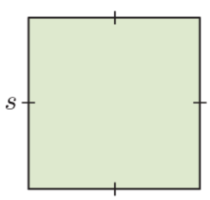
The **perimeter** of a figure is the distance around its boundary.

For a **polygon**, the perimeter is obtained by adding the lengths of all of its sides.

For a **circle**, the perimeter has the special name **circumference**.

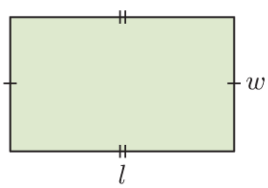
The following are perimeter formulae for commonly occurring shapes:

Square



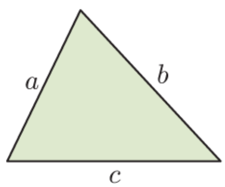
$P = 4s$

Rectangle



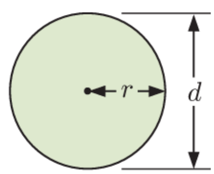
$P = 2(l + w)$

Triangle



$P = a + b + c$

Circle

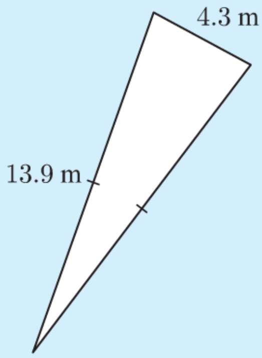


$C = 2\pi r$ or $C = \pi d$

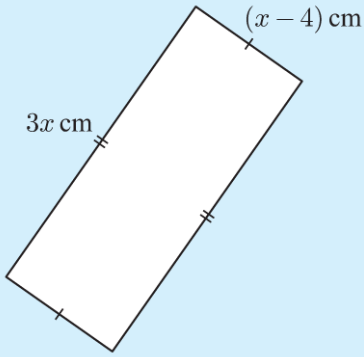
Example 7 **Self Tutor**

Find the perimeter of:

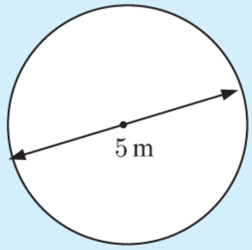
a



b



c



a $P = 4.3 + 2 \times 13.9$ m
$= 32.1$ m

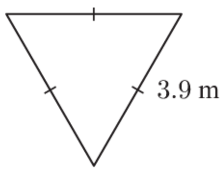
b $P = 2 \times 3x + 2 \times (x-4)$ cm
$= 6x + 2x - 8$ cm
$= 8x - 8$ cm

c $C = \pi d$
$= \pi \times 5$ m
≈ 15.7 m

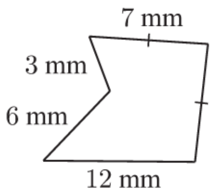
EXERCISE 3B

1 Find the perimeter of the following figures:

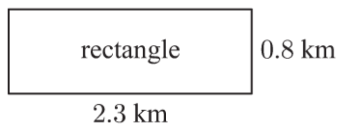
a



b

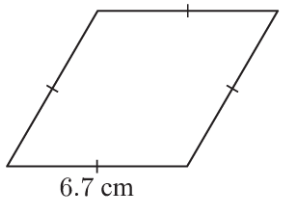


c

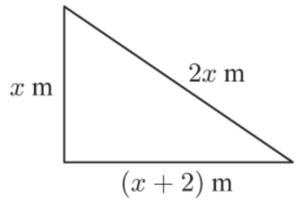


d

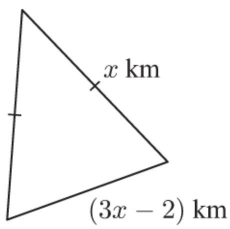
e



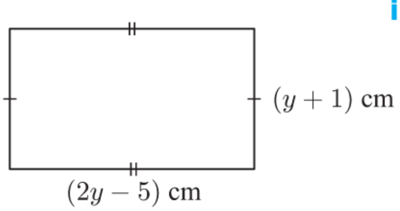
f



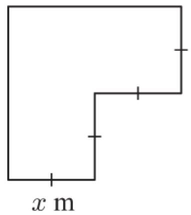
g



h



i



2 A rectangular field 220 metres long and 300 metres wide is surrounded by a post and rail fence.

a Draw and label a diagram of the field.

b Find the total length of the fence.

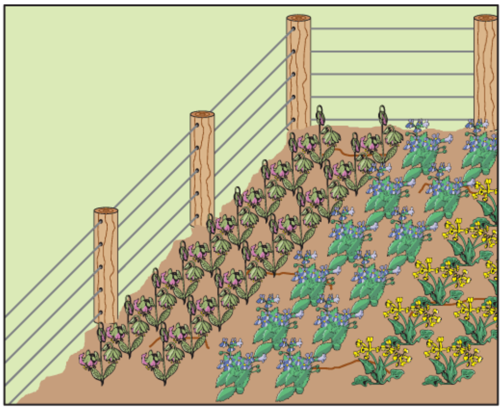
3 A farmer needs to fence his vegetable garden to keep out his sheep. The garden measures 26 m × 40 m. The fence has 5 strands of wire, and posts are placed every 2 metres. A gate occupies one of the 2 m gaps.

a Calculate the perimeter of the garden.

b What length of wire is needed?

c How many posts are needed?

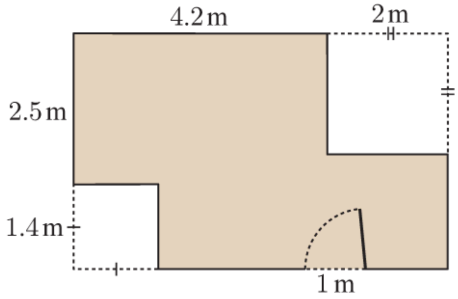
d Wire costs \$0.34 per metre, each post costs \$11.95, and the gate costs \$88. Find the total cost of the fence.



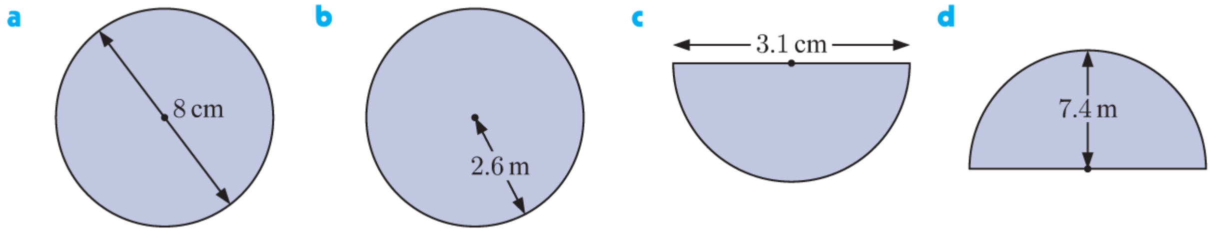
4 A room has the shape and dimensions shown. Skirting board must be laid around the edge of the room. It is sold in 2.4 metre lengths which can be cut as needed. Each length costs \$2.48.

a What length of skirting board is needed?

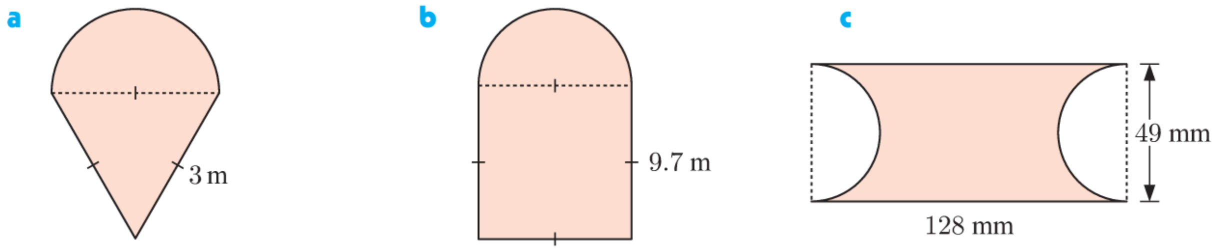
b Find the cost of the skirting board.



5 Find, to 4 significant figures, the perimeter of:



6 Find the perimeter of:

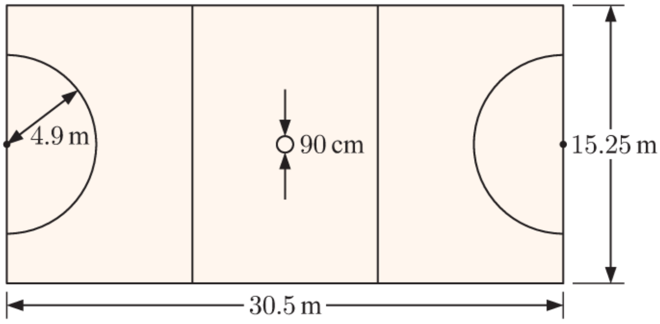


7 My bike wheels have radius 32 centimetres. When I ride to school, the wheels complete 639 revolutions. Calculate how far I live from my school, to the nearest metre.

8 A netball court has the dimensions shown.

a Find the perimeter of the court.

b Find the total length of all the marked lines.



Example 8 **Self Tutor**

Find the radius of a circle with circumference 11.7 m.

$$C = 11.7$$
$$\therefore \quad 2\pi r = 11.7$$
$$\therefore \quad r = \frac{11.7}{2\pi} \qquad \{\text{dividing both sides by } 2\pi\}$$
$$\therefore \quad r \approx 1.86$$

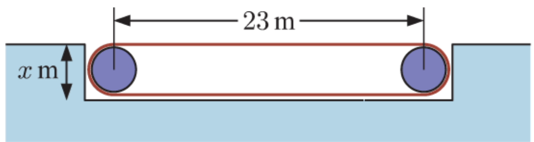
$\therefore$ the radius is approximately 1.86 m.

9 Find:

a the diameter of a circular pond with circumference 37.38 m

b the radius of a circular gazebo with circumference 32.67 m.

10 A conveyor belt 50 m long is used to carry objects a distance of 23 m. What depth is the recess needed to exactly contain the belt rollers?

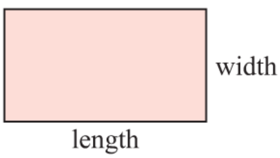


C AREA

The **area** of a closed figure is the number of square units it contains.

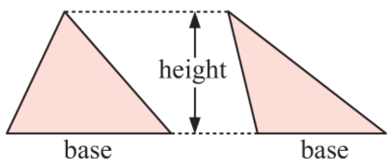
The following are area formulae for commonly occurring shapes:

Rectangle



Area = length × width

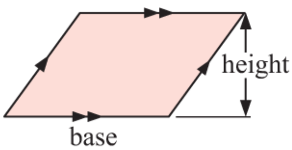
Triangle



DEMO

Area = $\frac{1}{2}$ × base × height

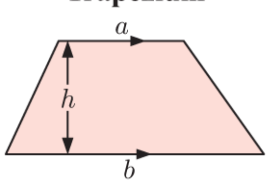
Parallelogram



DEMO

Area = base × height

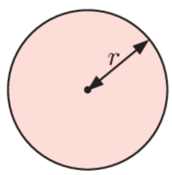
Trapezium



DEMO

$$\textbf{Area} = \left(\frac{a+b}{2}\right) \times h$$

Circle



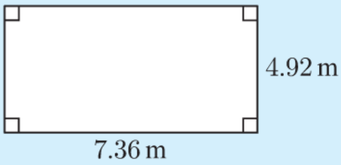
DEMO

Area = πr^2

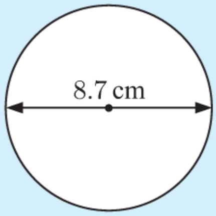
Example 9 **Self Tutor**

Find, to 4 significant figures, the area of:

a



b



a $A = lw$

$= 7.36 \times 4.92\ \text{m}^2$

$\approx 36.21\ \text{m}^2$

b The diameter $d = 8.7$ cm,

so the radius $r = \dfrac{8.7}{2} = 4.35$ cm

$\therefore\ A = \pi r^2$

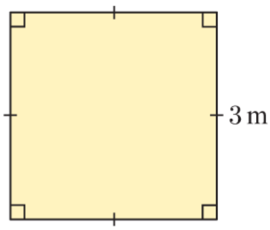
$= \pi \times (4.35)^2\ \text{cm}^2$

$\approx 59.45\ \text{cm}^2$

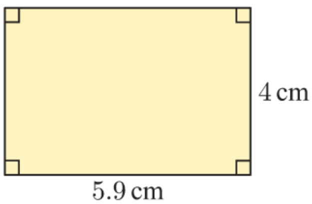
EXERCISE 3C

1 Find the area of:

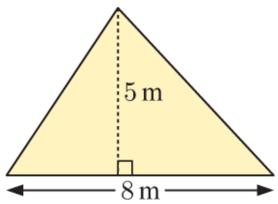
a



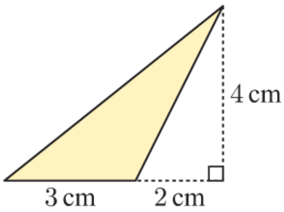
b



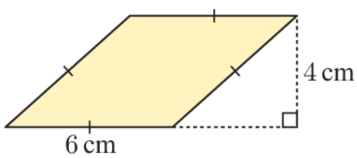
c



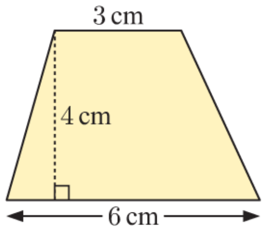
d



e

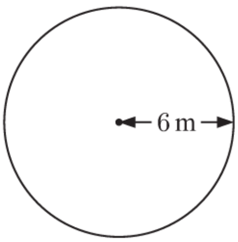


f

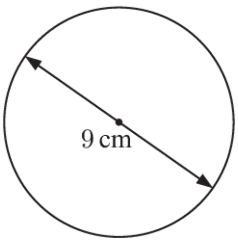


2 Find, to 1 decimal place, the area of:

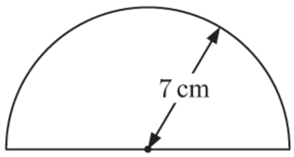
a



b

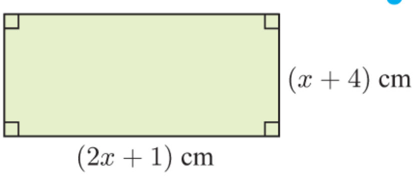


c

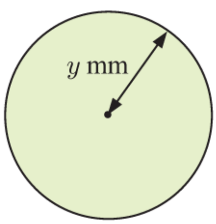


3 Write an expression for the area of each shape:

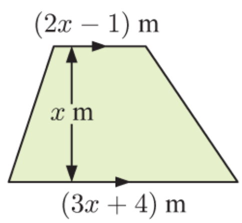
a



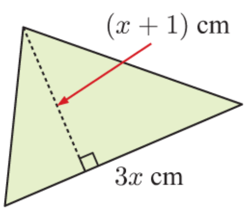
b



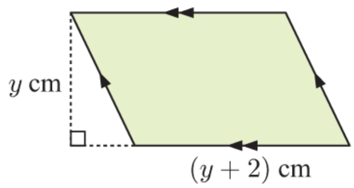
c



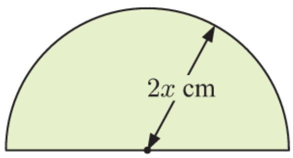
d



e



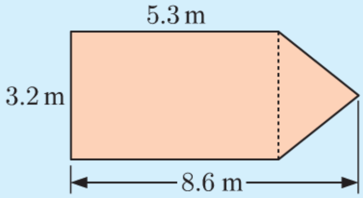
f



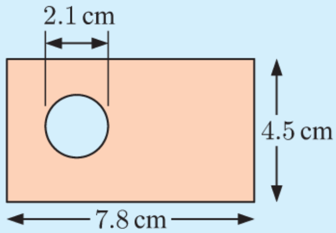
Example 10 **Self Tutor**

Calculate the shaded area:

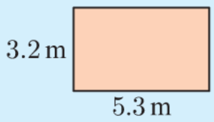
a



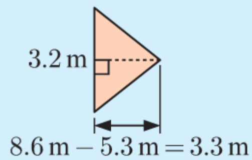
b



a The required area is the sum of



and



$\therefore$ the total area = area of rectangle + area of triangle

$= (5.3 \times 3.2) + (\frac{1}{2} \times 3.2 \times 3.3)\ \text{m}^2$

$= 22.24\ \text{m}^2$

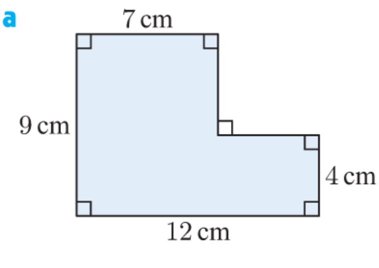
b Area = area of rectangle − area of circle

$= 7.8 \times 4.5 - \pi \times \left(\frac{2.1}{2}\right)^2\ \text{cm}^2$

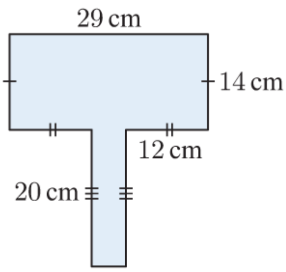
$\approx 31.6\ \text{cm}^2$

4 Calculate the area of each shape:

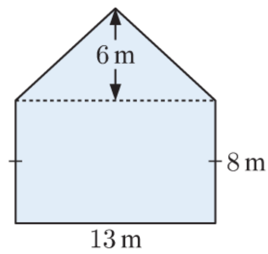
a



b

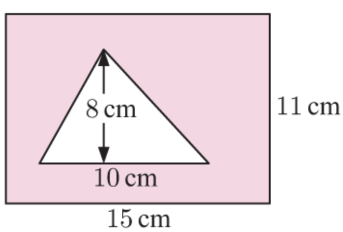


c

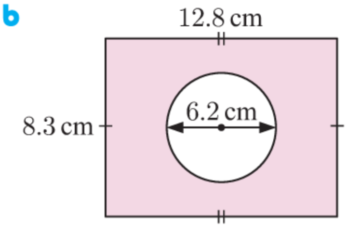


5 Calculate the area of the following shaded regions:

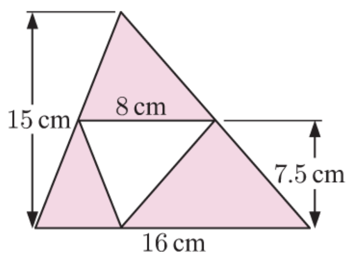
a



b

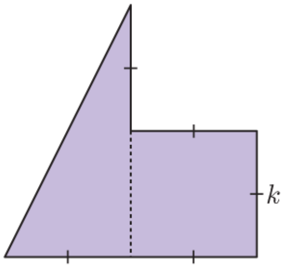


c

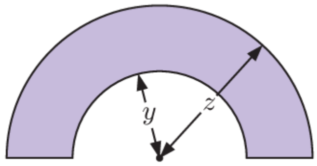


6 Write a formula for the area A of each shaded region:

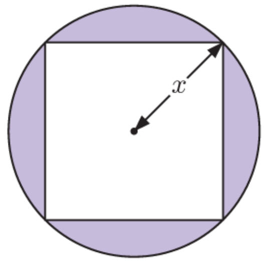
a



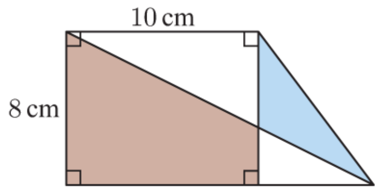
b



c

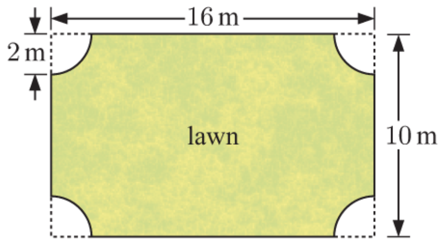


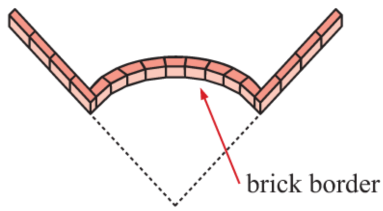
7 Given that the blue region has area 15 cm^2, find the area of the brown region.



8 Pete's Pizza Bar sells three sizes of circular pizza. The small pizza has diameter 15 cm and costs \$8.40. The medium size has diameter 30 cm and costs \$19.20. The super size has diameter 35 cm and costs \$28. Which of the pizzas gives the best value for money?

9 Gavin is landscaping his backyard. He wants to have a section of lawn with the dimensions shown below, with bricks around its border.





The bricks are 22 cm long, and cost \$4.70 each. The lawn costs \$15 per square metre.

a Find the length of the border.

b How many bricks will Gavin need?

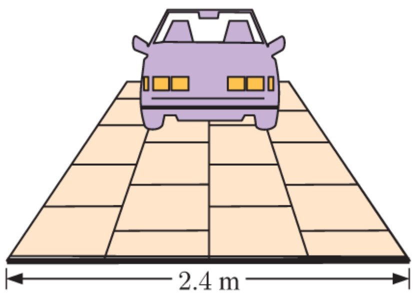
c Find the area of the lawn.

d Find the total cost of the project.

10 Concrete slabs suitable for a driveway are sold in two sizes:

Size A: 0.6 m by 0.6 m
Size B: 0.6 m by 0.3 m.

Both types of slab cost \$18.25 per square metre.
A driveway is to be 2.4 m wide and 18 m long. The slabs are laid on sand which costs \$18 per tonne. A tonne of sand covers 17.5 m^2 to the required depth. The sand must be purchased in a multiple of 0.2 of a tonne.



a Calculate the area of the driveway.

b Explain why you need 4 size B slabs for the pattern given in the diagram.

c How many size A slabs are needed?

d How much sand must be purchased?

e Find the total cost of the slabs and sand.

D SPEED

The most common rate that we use is **speed**, which is a comparison between the *distance travelled* and the *time taken*.

The **instantaneous speed** of an object refers to how fast the object is travelling at a given point in time.

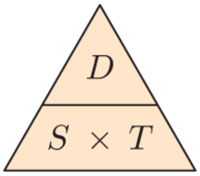
For example, when you are in a car, the speedometer might say that you are travelling at 50 km per hour.

However, when we go on a journey, we do not always travel at a constant speed. We need to slow down for other cars, and stop at traffic lights. For the whole journey, therefore, we calculate an **average speed** by comparing the total distance travelled with the total time taken.

$$\textbf{average speed} = \frac{\textbf{total distance travelled}}{\textbf{total time taken}}$$

This formula can be rearranged as: distance = speed × time

or $\text{time} = \dfrac{\text{distance}}{\text{speed}}$



You can use the triangle alongside to help you remember these.

Example 11 Self Tutor

A tour bus travels 238 km in 2 h 48 min.

a Find the average speed of the bus.

b Travelling with the same speed, how long would the bus take to complete a 761 km trip?

a
$$\begin{aligned}\text{average speed} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{238\text{ km}}{2\text{ h }48\text{ min}}\\ &= \frac{238\text{ km}}{2.8\text{ h}}\\ &= 85\text{ km h}^{-1}\end{aligned}$$

b
$$\begin{aligned}\text{time} &= \frac{\text{distance}}{\text{speed}}\\ &= \frac{761\text{ km}}{85\text{ km h}^{-1}}\\ &\approx 8.95\text{ h}\\ &\approx 8\text{ h }57\text{ min}\end{aligned}$$

EXERCISE 3D

1 Find, in km h^{-1}, the average speed of:

a a plane which flies 3200 km in 3 h 24 min

b a cyclist who rides 143 km in 5 h 17 min.

2 Find, in m s^{-1}, the average speed of:

a a runner who sprints 200 m in 24.38 s

b an arrow which flies 58 m in 0.87 s.

3 Find the time taken for:

- **a** a train to travel 184 km with average speed 128 $km\,h^{-1}$
- **b** a car to travel 33.2 km with average speed 47.5 $km\,h^{-1}$
- **c** a drone to fly 850 m with average speed 14.3 $m\,s^{-1}$
- **d** a lifesaver to swim 230 m with average speed 0.82 $m\,s^{-1}$.

4 Find the distance that:

- **a** a plane flying at 360 $km\,h^{-1}$ will travel in 2 h 20 min
- **b** a cyclist riding at 22.4 $km\,h^{-1}$ will travel in 46 min
- **c** a mosquito flying at 0.58 $m\,s^{-1}$ will travel in 35 s
- **d** a family walking at 1.6 $m\,s^{-1}$ will travel in 15 min.

5 A Caribbean yacht race takes place on a 148 nautical mile course. The winner finishes in 5 h 36 min 2 s.

- **a** Write the length of the race in km.
- **b** Write the winning time as a decimal number of hours.
- **c** Find the average speed of the winning yacht.

Background Knowledge 4

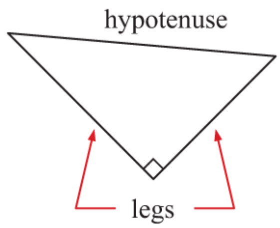
Pythagoras' theorem

Contents:

A PYTHAGORAS' THEOREM

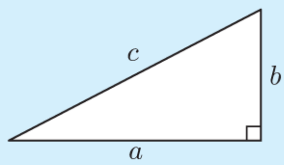
A **right angled triangle** is a triangle which has a right angle in one corner.

The side opposite the right angle is called the **hypotenuse**, and is the longest side of the triangle. The other two sides are called the **legs** of the triangle.



Pythagoras' theorem is:

> In a right angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
>
> $$c^2 = a^2 + b^2$$

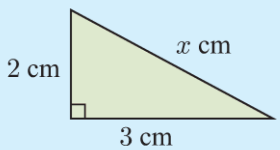


If we know the lengths of any two sides of a right angled triangle, we can use Pythagoras' theorem to find the third side.

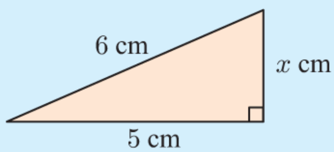
Example 1 **Self Tutor**

Find the unknown length:

a



b



a The hypotenuse is opposite the right angle and has length x cm.

$\therefore\ x^2 = 3^2 + 2^2$ {Pythagoras}

$\therefore\ x^2 = 9 + 4$

$\therefore\ x^2 = 13$

$\therefore\ x = \sqrt{13}$ {as $x > 0$}

So, the hypotenuse is $\sqrt{13}$ cm long.

b The hypotenuse has length 6 cm.

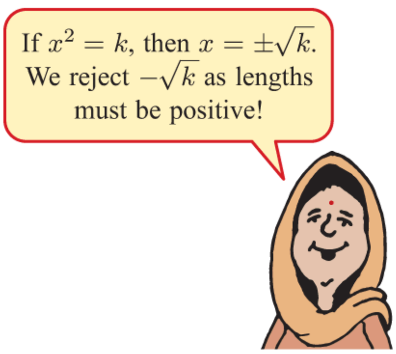
$\therefore\ x^2 + 5^2 = 6^2$ {Pythagoras}

$\therefore\ x^2 + 25 = 36$

$\therefore\ x^2 = 11$

$\therefore\ x = \sqrt{11}$ {as $x > 0$}

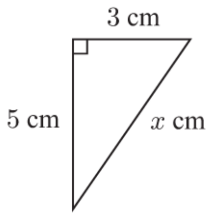
So, the third side is $\sqrt{11}$ cm long.



EXERCISE 4A

1 Find the length of the hypotenuse in the following right angled triangles.

a



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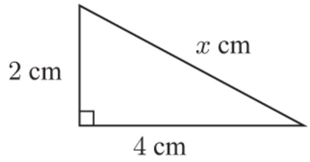

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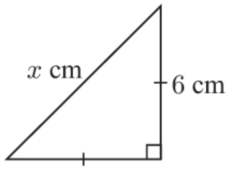
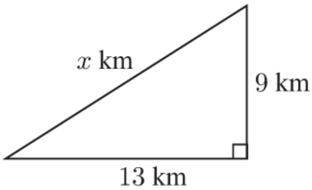
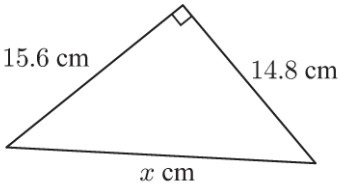


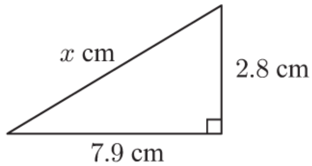
d



e

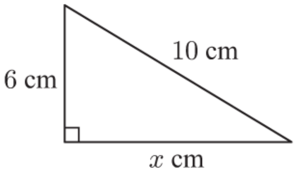


f

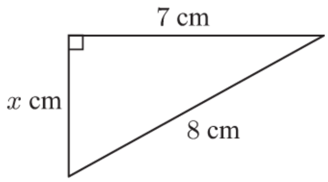


2 Find the length of the third side in the following right angled triangles.

a



b



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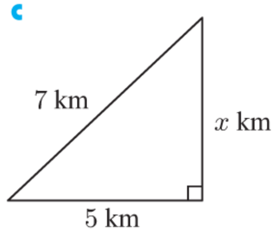

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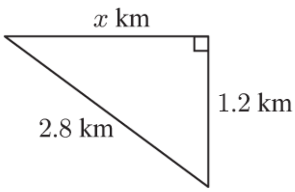

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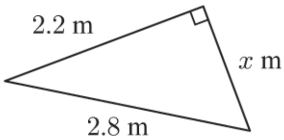

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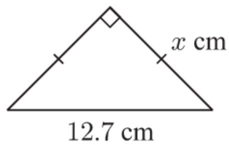
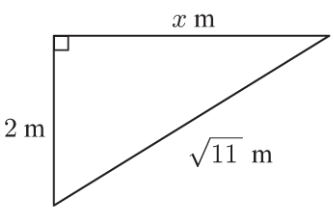


3 Solve for x:

a



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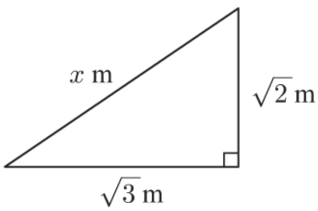

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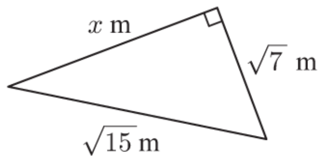

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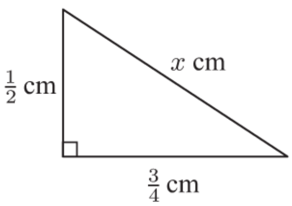

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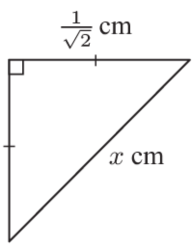

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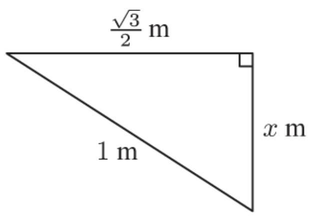
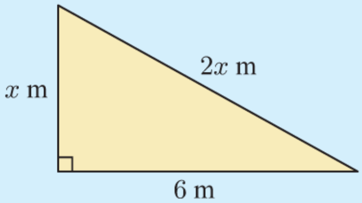


Example 2 — Self Tutor

Find the value of x:



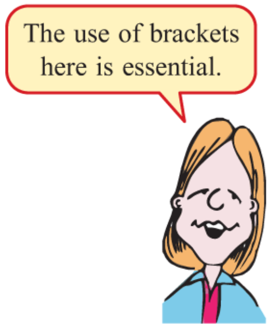
$(2x)^2 = x^2 + 6^2$ {Pythagoras}

$\therefore\ 4x^2 = x^2 + 36$

$\therefore\ 3x^2 = 36$

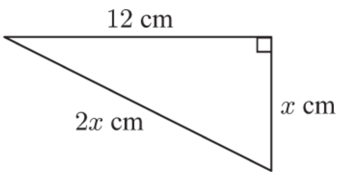
$\therefore\ x^2 = 12$

$\therefore\ x = \sqrt{12}$ {as $x > 0$}



4 Find the value of x:

a



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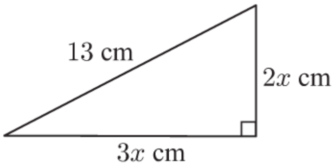

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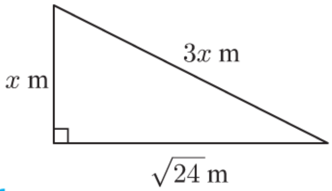

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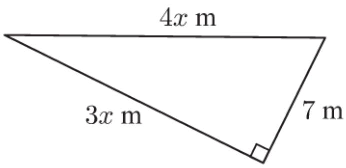

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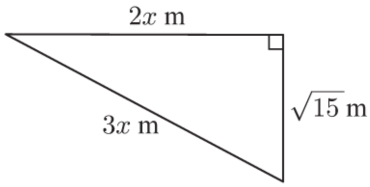

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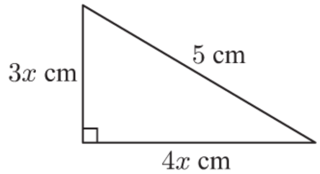
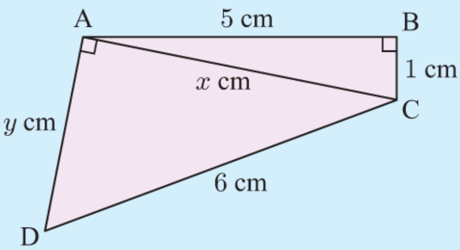


Example 3 — Self Tutor

Find the values of x and y:



In $\triangle$ABC, the hypotenuse is x cm long.

$\therefore\ x^2 = 5^2 + 1^2$ {Pythagoras}

$\therefore\ x^2 = 26$

$\therefore\ x = \sqrt{26}$ {as $x > 0$}

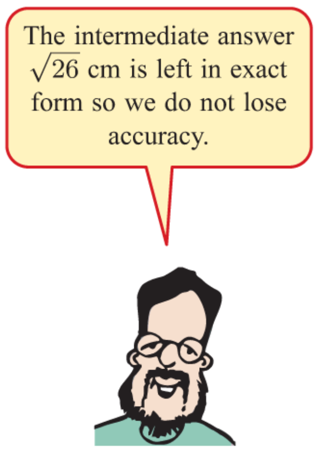
In $\triangle$ACD, the hypotenuse is 6 cm long.

$\therefore\ y^2 + (\sqrt{26})^2 = 6^2$ {Pythagoras}

$\therefore\ y^2 + 26 = 36$

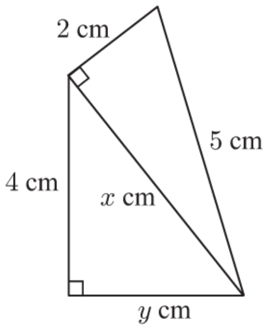
$\therefore\ y^2 = 10$

$\therefore\ y = \sqrt{10}$ {as $y > 0$}



5 Find the values of x and y:

a



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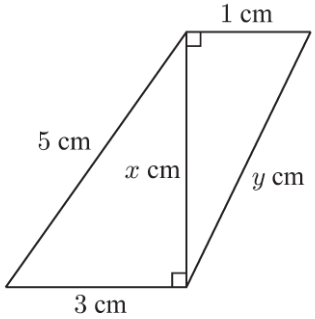

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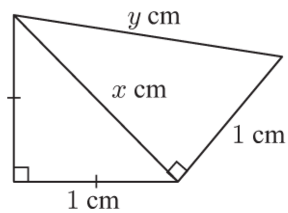
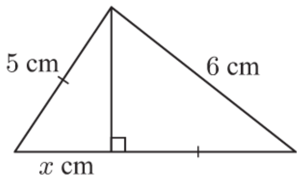


6 Find the value of x:

a



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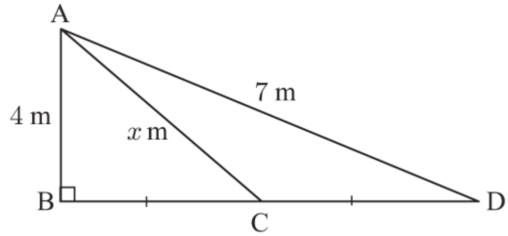
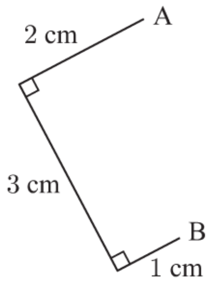


7 In the following figures, draw additional lines to complete right angled triangles. Apply Pythagoras' theorem to find the unknown distance AB.

a



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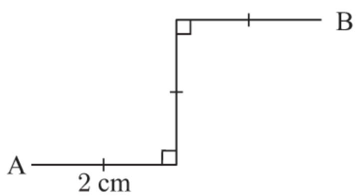

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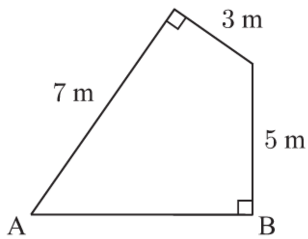
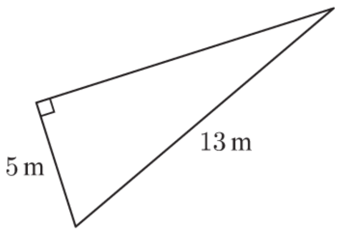


8 Find the area of each triangle:

a



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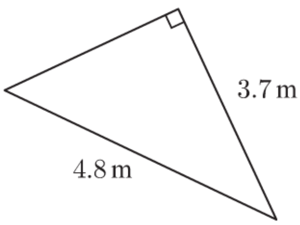


B THE CONVERSE OF PYTHAGORAS' THEOREM

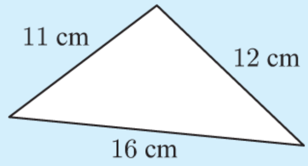
If we know all the side lengths of a triangle, we can determine whether the triangle is right angled by using the **converse of Pythagoras' theorem:**

> If a triangle has sides of length a, b, and c units and $a^2 + b^2 = c^2$, then the triangle is right angled.

GEOMETRY PACKAGE

Example 4 **Self Tutor**

The dimensions marked on this triangle are correct, but the triangle is not drawn to scale. Is it a right angled triangle?



The two shorter sides have lengths 11 cm and 12 cm.

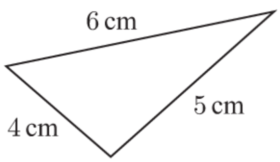
Now $11^2 + 12^2 = 121 + 144 = 265$

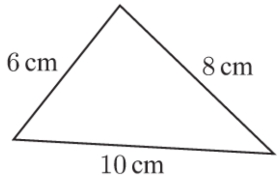
whereas $16^2 = 256$

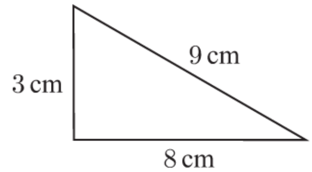
Since $11^2 + 12^2 \neq 16^2$, the triangle is not right angled.

EXERCISE 4B

1 The following triangles are not drawn to scale, but the information on them is correct. Which of the triangles is right angled?

a


b


c


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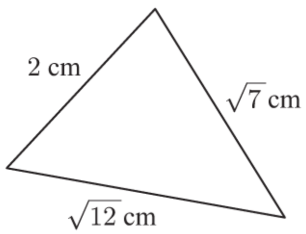

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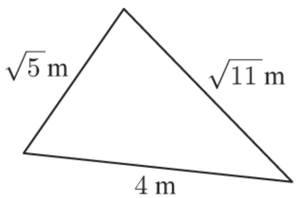

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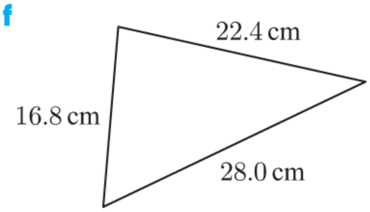
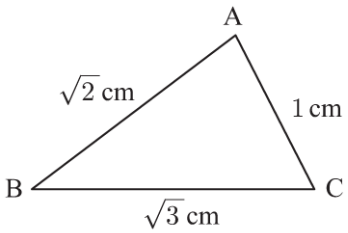
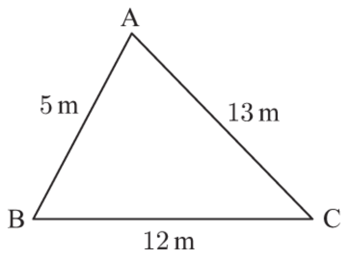
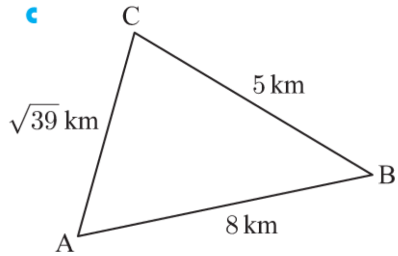


2 The following triangles are not drawn to scale, but the information on them is correct. If any of them is right angled, find the vertex where the right angle occurs.

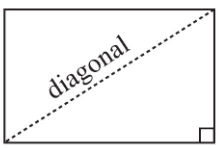
a


b


c


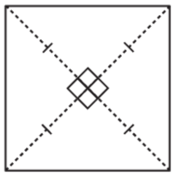
C RIGHT ANGLES IN GEOMETRIC FIGURES

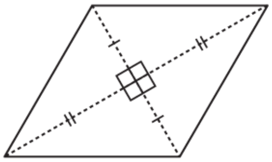
There are many geometric figures which involve right angles. It is important to recognise these because it tells us when we can apply Pythagoras' theorem.



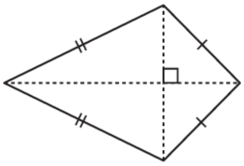
rectangle

In a **rectangle**, right angles exist between adjacent sides. We can construct a diagonal to form a right angled triangle.


square


rhombus

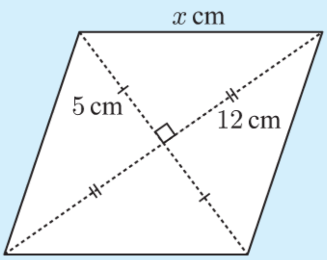
In a **square** and a **rhombus**, the diagonals bisect each other at right angles.


kite

In a **kite**, the diagonals intersect at right angles.

Example 5 **Self Tutor**

A rhombus has diagonals of length 10 cm and 24 cm. Find the lengths of its sides.



The diagonals of a rhombus bisect at right angles.

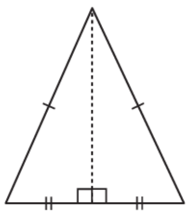
Let each side of the rhombus have length x cm.

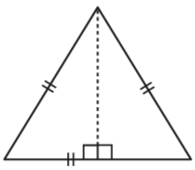
$\therefore\ x^2 = 5^2 + 12^2$ {Pythagoras}

$\therefore\ x^2 = 169$

$\therefore\ x = 13$ {as $x > 0$}

The sides are 13 cm long.


isosceles triangle


equilateral triangle

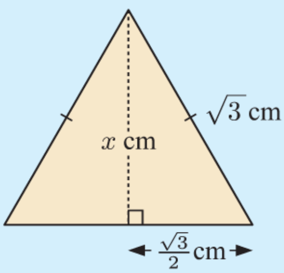
In an **isosceles triangle** and an **equilateral triangle**, the altitude bisects the base at right angles.

Example 6 **Self Tutor**

a Find the altitude of an equilateral triangle with sides $\sqrt{3}$ cm long.

b Hence find the area of the triangle.

a



Let the altitude be x cm.

The altitude bisects the base at right angles.

$\therefore\ x^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = (\sqrt{3})^2$ {Pythagoras}

$\therefore\ x^2 + \frac{3}{4} = 3$

$\therefore\ x^2 = \frac{9}{4}$

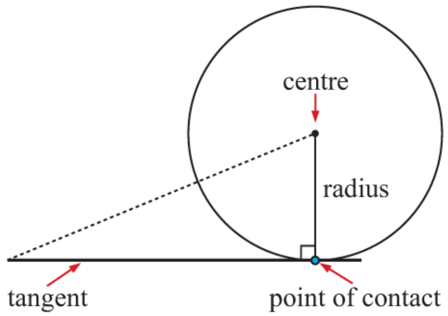
$\therefore\ x = \frac{3}{2}$ {as $x > 0$}

So, the altitude of the triangle is $\frac{3}{2}$ cm long.

b Area $= \frac{1}{2} \times \text{base} \times \text{height}$

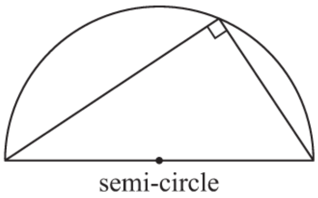
$= \frac{1}{2} \times \sqrt{3} \times \frac{3}{2}$

$\approx 1.30\ \text{cm}^2$

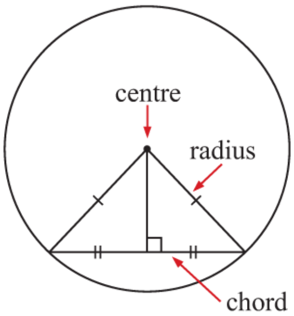


A **tangent** to a circle and a radius at the point of contact meet at right angles.

We can form a right angled triangle with another point on the tangent.



The angle in a **semi-circle** is always a right angle.



The line drawn from the centre of a circle at right angles to a **chord** bisects the chord.

Example 7 **Self Tutor**

A circle with radius 5 cm has a chord of length 4 cm.
Find the shortest distance from the centre of the circle to the chord.

The shortest distance is the "perpendicular distance".

The line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.

$\therefore$ AB = BC = 2 cm

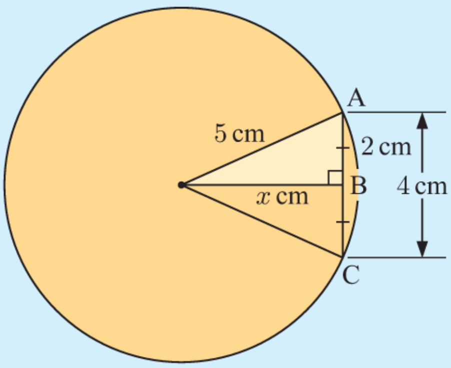
In $\triangle$AOB, $x^2 + 2^2 = 5^2$ {Pythagoras}

$\therefore x^2 = 25 - 4 = 21$

$\therefore x = \sqrt{21}$ {as $x > 0$}

$\therefore x \approx 4.58$

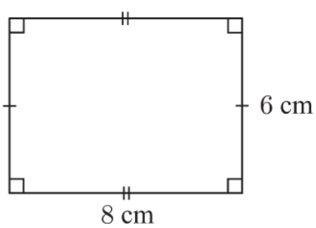
So, the shortest distance is about 4.58 cm.



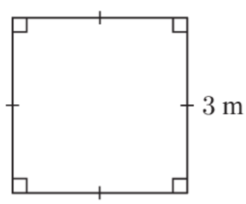
EXERCISE 4C

1 Find the lengths of the diagonals of these rectangles:

a



b



2 Find the lengths of the diagonals of a 12 mm $\times$ 16 mm rectangle.

3 The shorter side of a rectangle is 5 mm, and its diagonal is 11 mm. Find:

a the length of the longer side **b** the area of the rectangle.

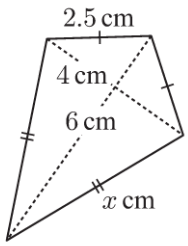
4 The longer side of a rectangle is three times the length of the shorter side. The diagonal has length $\sqrt{1000}$ m. Find the exact dimensions of the rectangle.

5 A rhombus has diagonals of length 2 cm and 4 cm. Find the length of its sides in surd form.

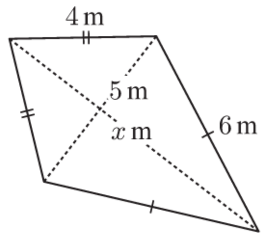
6 Find the side length of a square with diagonals 18 cm long.

7 Find the value of x:

a



b



8 A rhombus has sides of length 8 m. Its shorter diagonal is 10 m long. Find:

a the length of the longer diagonal **b** the area of the rhombus.

9 An isosceles triangle has equal sides of length 6 cm. Its third side is 8 cm long. Find:

a the altitude of the triangle **b** the area of the triangle.

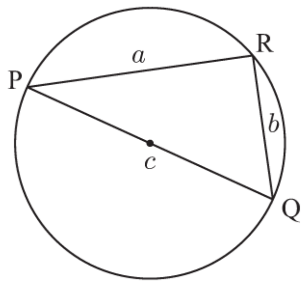
10 The base of an isosceles triangle is 6 cm long, and its area is 12 cm^2. Find the length of the two equal sides.

11 The altitude of an equilateral triangle is $2\sqrt{3}$ mm in length. Find the perimeter of the triangle.

12 Consider the circle alongside with diameter PQ.

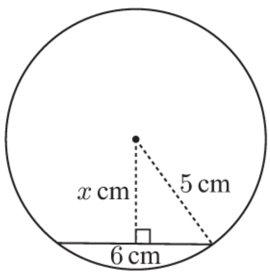
a What is the measure of $\text{P}\widehat{\text{R}}\text{Q}$?

b Write down an equation relating a, b, and c.

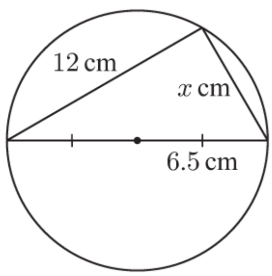


13 Find x:

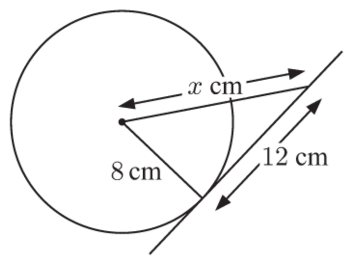
a



b

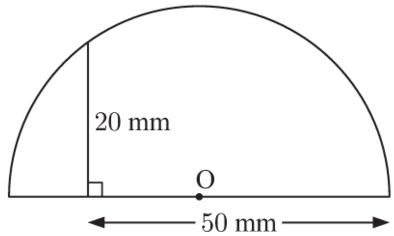


c



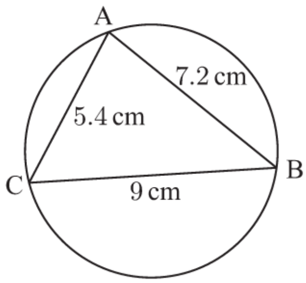
14 The chord of a circle is 8 cm long. The closest point of the chord to the centre of the circle is 3 cm away from it. Find the radius of the circle.

15 Find the radius of this semi-circle.

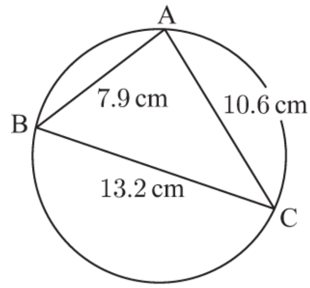


16 Triangle ABC is drawn using three points on a circle. Determine whether BC is a diameter of the circle.

a



b



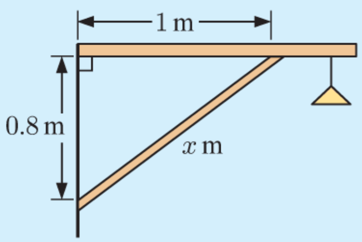
D PROBLEM SOLVING

Right angled triangles occur frequently in **problem solving**. Any time a right angle is present you should check whether Pythagoras' theorem can be used and whether it is beneficial.

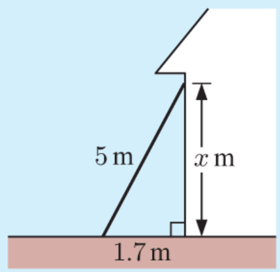
Example 8 Self Tutor

Find the unknown length in:

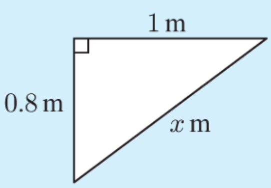
a



b



a



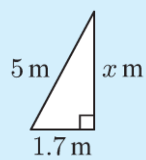
$x^2 = 0.8^2 + 1^2$ {Pythagoras}

$\therefore\ x^2 = 1.64$

$\therefore\ x \approx 1.28$ {as $x > 0$}

So, the length is about 1.28 m.

b



$x^2 + 1.7^2 = 5^2$ {Pythagoras}

$\therefore\ x^2 + 2.89 = 25$

$\therefore\ x^2 = 22.11$

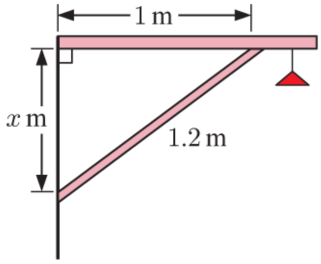
$\therefore\ x \approx 4.70$ {as $x > 0$}

So, the length is about 4.70 m.

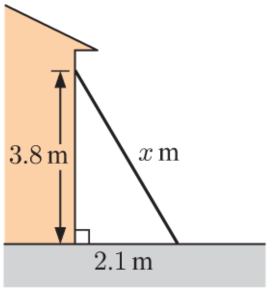
EXERCISE 4D

1 Find the value of x, correct to 3 significant figures:

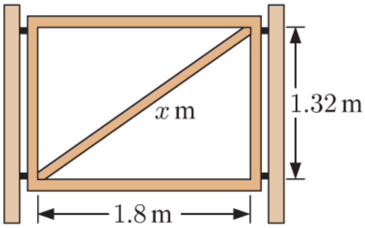
a



b

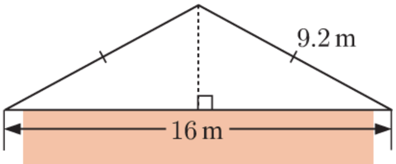


c

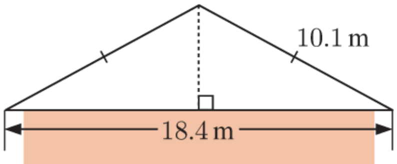


2 How high is the roof above the walls in the following roof structures?

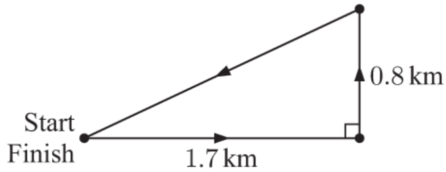
a



b



3 A sailing crew races 4 laps around the course shown. What distance do they travel in total?



4 A schooner sails to a point 46 km north and 74 km east of its port. How far is the ship from its port?

5 A runner is 2.2 km east and 1.5 km south of her starting point.

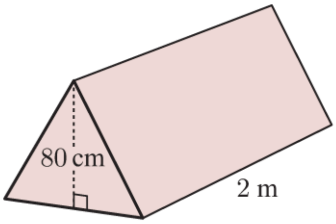
a How far is she from her starting point?

b If she can run at 10 km h^{-1}, how long will it take her to return to her starting point in a direct line?

6 Rohan is building a rabbit hutch in the shape of an equilateral triangular prism. Its height is 80 cm.

a How long are the sides of the triangle?

b Find the area of ground covered by the rabbit hutch.



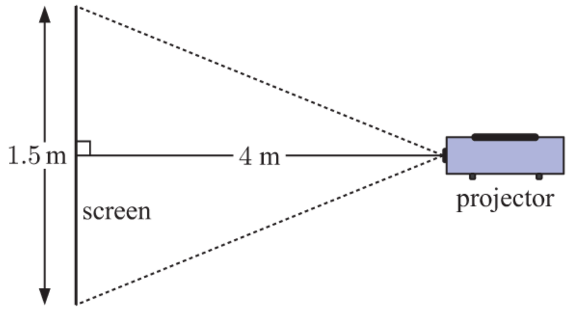
7

Captain Jack and Captain Will leave Bridgetown at the same time. Jack sails due east at a constant speed of 15 km h^{-1}, and Will sails due south at a constant speed of 19 km h^{-1}.

a How far has each captain travelled after two hours?

b Find the distance between the captains 2 hours after leaving Bridgetown.

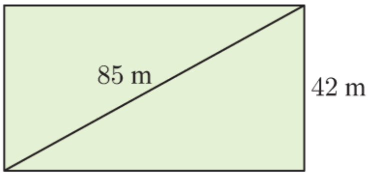
8 A projector is 4 m from the middle of a screen 1.5 m high. How much further away from the projector is the top edge than the centre of the screen?



9 Julia walks to school across the diagonal of a rectangular field. The diagonal is 85 metres long. One side of the field is 42 m.

a Find the length of the other side of the field.

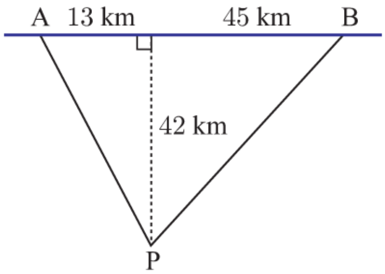
b By how much is it shorter to walk across the diagonal, than around two sides?



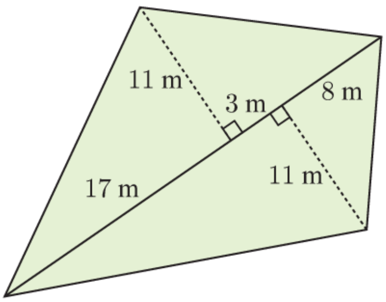
10 A power station P supplies two towns A and B with electricity. New underground power lines to each town are required. The towns are connected by a straight highway through A and B, and the power station is 42 km from this highway.

a Find the length of power line required from P to each town.

b Find the total cost of the new power line given that each kilometre will cost $23 500.

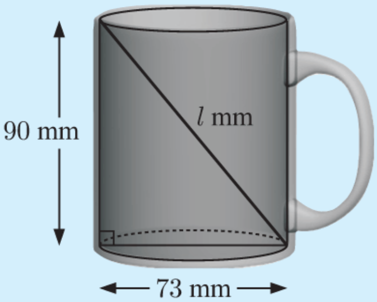


11 The diagram shows various measurements of a field. Calculate the perimeter of the field, to the nearest metre.



Example 9 — Self Tutor

Michael's coffee mug is 90 mm high and 73 mm in diameter. It is filled to the brim with steaming hot coffee. Michael does not like sugar, but he always stirs in his milk. What is the minimum length stirrer Michael needs so that if he drops it in, it will not disappear in the coffee?



Let the stirrer have length l mm.

If the stirrer fits exactly in the mug, we have a right angled triangle.

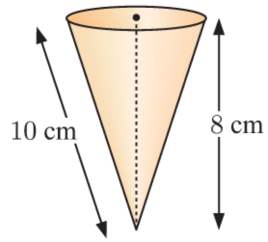
By Pythagoras, $l^2 = 73^2 + 90^2$

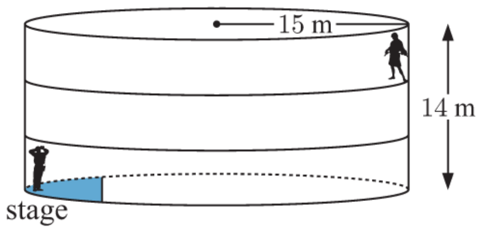
$\therefore\ l = \sqrt{(73^2 + 90^2)}$ {as $l > 0$}

$\therefore\ l \approx 115.9$ mm

So, the stirrer must be at least 116 mm long.

12 An ice cream cone is 8 cm tall and its slant height is 10 cm. Find the radius of the circle at the top of the cone.



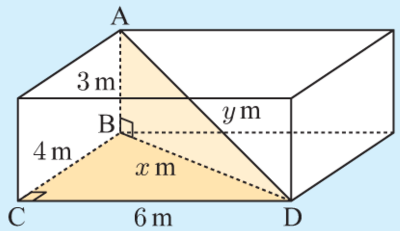
13 

An actor stands at the back of the stage of Shakespeare's Globe theatre, which is cylindrical.
How far must his voice reach so that he can be heard by the audience member furthest away from him?

14 A cylindrical soft drink can is 8 cm wide and 12 cm high, with a hole in the middle of the top for the straw. How long must the straw be so that all of the soft drink can be reached, and there is 2 cm of straw sticking out at the top?

Example 10 Self Tutor

The floor of a room is 6 m by 4 m and has height 3 m. Find the distance from a corner point on the floor to the opposite corner point on the ceiling.



We label corners of the room as shown.

The required distance is AD.

In $\triangle$BCD, $x^2 = 4^2 + 6^2$ {Pythagoras}

$\therefore\ x^2 = 16 + 36 = 52$

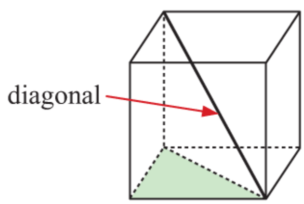
In $\triangle$ABD, $y^2 = x^2 + 3^2$

$\therefore\ y^2 = 52 + 9 = 61$

$\therefore\ y = \sqrt{61} \approx 7.81$

So, the distance from a corner point on the floor to the opposite corner point on the ceiling is about 7.81 m.

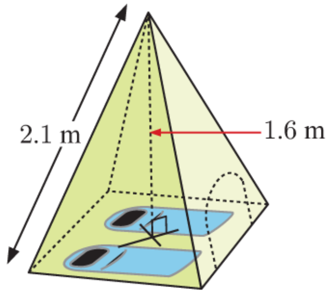
15 A cube has sides of length 10 cm.
Find the length of a diagonal of the cube.



16 A room is 7 m by 4 m and has a height of 3 m. Find the distance from a corner point on the floor to the opposite corner point on the ceiling.

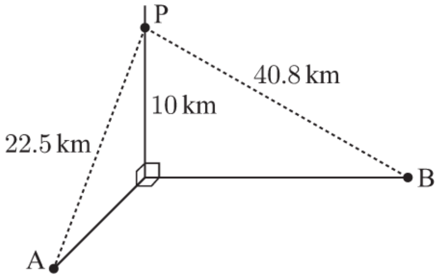
17 Skyways Airlines has the policy that passengers cannot carry on luggage with diagonal measurement of more than 56 cm. Katie's bag is 40 cm $\times$ 30 cm $\times$ 25 cm. Is she allowed to carry it on board the plane?

18 A square-based pyramid tent has a centre pole 1.6 m high, and edge poles 2.1 m long. What is the maximum possible height of a camper who sleeps along the edge of the tent?



19 A pyramid of height 40 m has a square base with edges of length 50 m. Determine the length of its slant edges.

20



An aeroplane P is flying at an altitude of 10 000 m. Ship A is due south and 22.5 km from P in a direct line. Ship B is due east and 40.8 km from P in a direct line. Find the distance between the two ships.

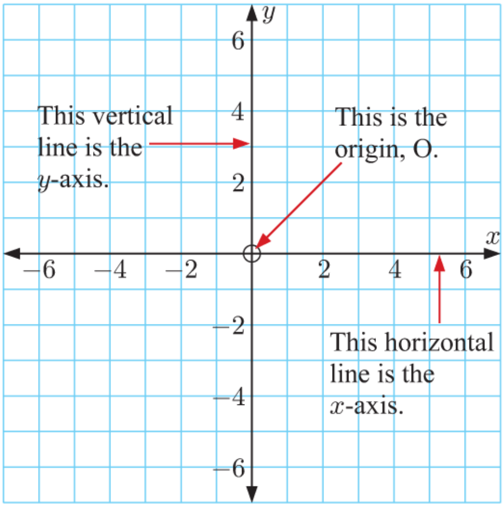
Background Knowledge 5

Coordinate geometry

Contents:

THE NUMBER PLANE

The number grid alongside is a **Cartesian plane**, named after **René Descartes**. The numbers or **coordinates** on it allow us to locate the exact position of any point on the plane.

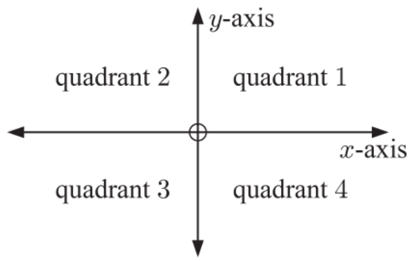


We start with a point of reference O called the **origin**. Through it we draw a horizontal line called the ***x*-axis**, and a vertical line called the ***y*-axis**.

The ***x*-axis** is an ordinary number line with positive numbers to the right of O and negative numbers to the left of O.

The ***y*-axis** has positive numbers above O and negative numbers below O.

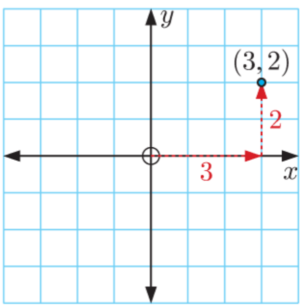
The axes divide the number plane into four **quadrants**.



PLOTTING POINTS ON THE CARTESIAN PLANE

To specify the position of a point on the number plane, we use an **ordered pair** of coordinates in the form (x, y).

For example, on the grid alongside we see the point described by the ordered pair of coordinates $(3, 2)$. We say that 3 is the ***x*-coordinate** and 2 is the ***y*-coordinate**.



A THE DISTANCE BETWEEN TWO POINTS

To go from $A(x_1, y_1)$ to $B(x_2, y_2)$,

the x-step $= x_2 - x_1$

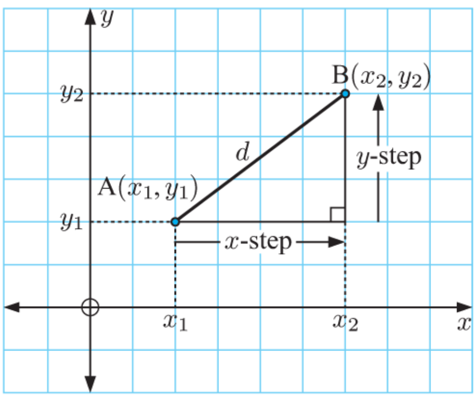
and the y-step $= y_2 - y_1$.

Using Pythagoras' theorem,

$$(\text{AB})^2 = (x\text{-step})^2 + (y\text{-step})^2$$

$$\therefore \text{AB} = \sqrt{(x\text{-step})^2 + (y\text{-step})^2}$$

$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The distance d between two points (x_1, y_1) and (x_2, y_2) is given by

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}.$$

Example 1 Self Tutor

Find the distance between A(6, 3) and B(8, −2).

$$\underset{x_1\ y_1}{\text{A}(6,\ 3)} \qquad \underset{x_2\ y_2}{\text{B}(8,\ -2)}$$

$$\begin{aligned} \text{AB} &= \sqrt{(8-6)^2 + (-2-3)^2} \\ &= \sqrt{2^2 + (-5)^2} \\ &= \sqrt{4 + 25} \\ &= \sqrt{29} \text{ units} \end{aligned}$$

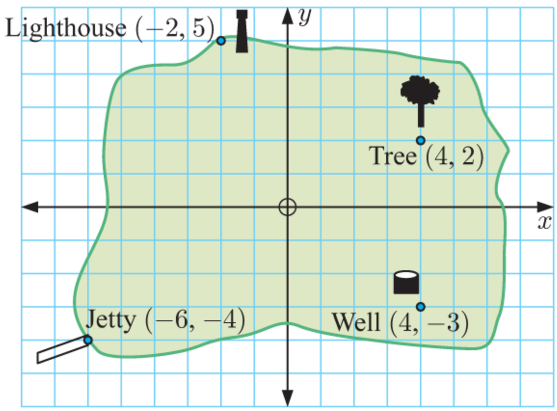
EXERCISE 5A

1 Find the distance from:

a A(2, 6) to B(3, 3)
b C(−2, 3) to D(1, 5)
c M(2, 4) to N(−1, −3)
d O(0, 0) to P(−2, 4)
e R(3, −2) to S(5, −2)
f T(0, 3) to U(2, −1)
g W(−4, 0) to X(0, 3)
h Y(−1, −4) to Z(−3, 3).

2 On the map alongside, each grid unit represents 1 km.
Find the distance between:

a the lighthouse and the tree
b the jetty and the lighthouse
c the well and the tree
d the lighthouse and the well.



Example 2 Self Tutor

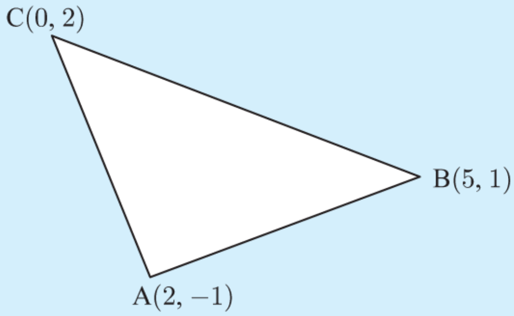
The points A(2, −1), B(5, 1), and C(0, 2) form a triangle ABC.

a Use the distance formula to classify the triangle as equilateral, isosceles, or scalene.
b Determine whether the triangle has a right angle.

a
$$\begin{aligned} \text{AB} &= \sqrt{(5-2)^2 + (1-(-1))^2} \\ &= \sqrt{3^2 + 2^2} \\ &= \sqrt{13} \text{ units} \end{aligned}$$

$$\begin{aligned} \text{AC} &= \sqrt{(0-2)^2 + (2-(-1))^2} \\ &= \sqrt{(-2)^2 + 3^2} \\ &= \sqrt{13} \text{ units} \end{aligned}$$

$$\begin{aligned} \text{BC} &= \sqrt{(0-5)^2 + (2-1)^2} \\ &= \sqrt{(-5)^2 + 1^2} \\ &= \sqrt{26} \text{ units} \end{aligned}$$



Since AB = AC but not BC, triangle ABC is isosceles.

b $\text{AB}^2 + \text{AC}^2 = 13 + 13$
$= 26$
$= \text{BC}^2$

Using the converse of Pythagoras' theorem, triangle ABC is right angled.
The right angle is at A, opposite the longest side.

3 Use the distance formula to classify triangle ABC as either equilateral, isosceles, or scalene.

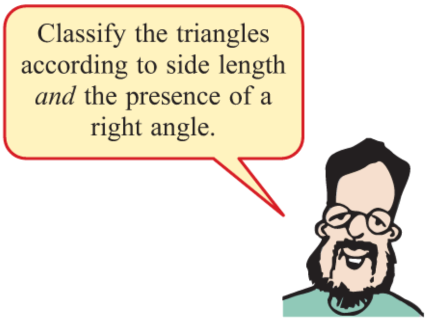
a A$(-1, 0)$, B$(-2, 3)$, C$(-5, 4)$
b A$(-2, -4)$, B$(1, 4)$, C$(2, -3)$
c A$(0, 1)$, B$(0, -1)$, C$(-\sqrt{3}, 0)$
d A$(0, -4)$, B$(\sqrt{3}, 1)$, C$(3\sqrt{3}, -5)$

4 Determine whether the following triangles are right angled. If there is a right angle, state the vertex where it is located.

a A$(1, -1)$, B$(-1, 2)$, C$(7, 3)$
b A$(-1, 2)$, B$(3, 4)$, C$(5, 0)$
c A$(-2, 3)$, B$(-5, 4)$, C$(1, 2)$
d A$(5, 4)$, B$(-4, 6)$, C$(-3, 2)$

5 Fully classify the triangle formed by the following points:

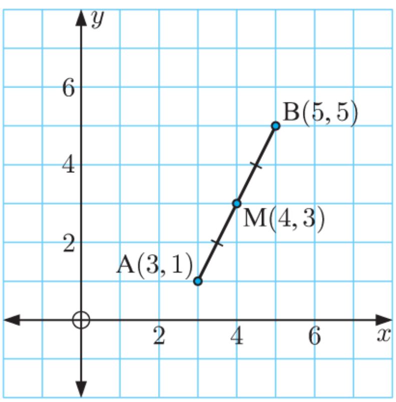
a A$(-4, 5)$, B$(3, 4)$, C$(8, -1)$
b A$(2, -5)$, B$(-2, 2)$, C$(-4, -1)$
c A$(-2, 1)$, B$(-3, 4)$, C$(1, 2)$
d A$(\sqrt{3}, -1)$, B$(0, 2)$, C$(-\sqrt{3}, -1)$



B MIDPOINTS

The **midpoint** of line segment [AB] is the point midway between points A and B.

Consider the points A$(3, 1)$ and B$(5, 5)$. M is at $(4, 3)$ on the line segment connecting A and B.



Using the distance formula,

$$\text{AM} = \sqrt{(4-3)^2 + (3-1)^2} = \sqrt{5} \text{ units, and}$$
$$\text{MB} = \sqrt{(5-4)^2 + (5-3)^2} = \sqrt{5} \text{ units.}$$

So, M is the midpoint of AB.

The x-coordinate of M is the *average* of the x-coordinates of A and B.

$$x_\text{M} = \frac{3+5}{2} = 4$$

The y-coordinate of M is the *average* of the y-coordinates of A and B.

$$y_\text{M} = \frac{1+5}{2} = 3$$

Given $A(x_1,\, y_1)$ and $B(x_2,\, y_2)$, the **midpoint** of [AB] has coordinates $\left(\dfrac{x_1+x_2}{2},\ \dfrac{y_1+y_2}{2}\right)$.

Example 3 — Self Tutor

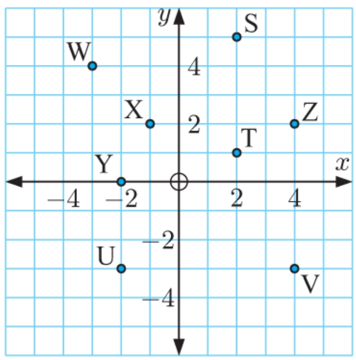
Given A(−1, 3) and B(5, −2), find the coordinates of the midpoint M of [AB].

The x-coordinate of M $= \dfrac{-1+5}{2} = \dfrac{4}{2} = 2$

The y-coordinate of M $= \dfrac{3+(-2)}{2} = \dfrac{1}{2}$

$\therefore$ the midpoint of [AB] is $M(2,\, \frac{1}{2})$.

EXERCISE 5B

1 

Using the diagram only, find the coordiantes of the midpoint of the line segment:

- **a** [ST]
- **b** [UV]
- **c** [WX]
- **d** [YZ]
- **e** [SV]
- **f** [UT]
- **g** [YT]
- **h** [TV]

2 Find the coordinates of the midpoint of the line segment joining:

- **a** (2, 5) and (4, 7)
- **b** (1, 6) and (4, 2)
- **c** (0, 3) and (2, 0)
- **d** (3, −2) and (3, 2)
- **e** (−1, 4) and (2, 2)
- **f** (0, −3) and (−2, 5)
- **g** (−4, −1) and (3, −2)
- **h** (1, 0) and (−6, 8).

Example 4 — Self Tutor

M is the midpoint of [AB], where A is (−1, 4) and M is (2, 3). Find the coordinates of B using:

- **a** the midpoint formula
- **b** equal steps.

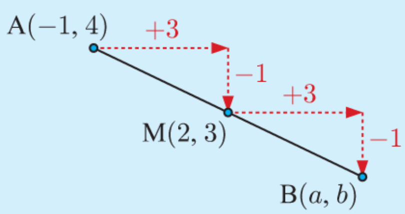
a Let B have coordinates $(a,\, b)$.

$$\therefore\ \frac{a+(-1)}{2} = 2 \quad\text{and}\quad \frac{b+4}{2} = 3$$

$$\therefore\ a - 1 = 4 \quad\text{and}\quad b + 4 = 6$$

$$\therefore\ a = 5 \quad\text{and}\quad b = 2$$

So, B is the point (5, 2).

b 

x-step: $-1 \xrightarrow{+3} 2 \xrightarrow{+3} 5$

y-step: $4 \xrightarrow{-1} 3 \xrightarrow{-1} 2$

$\therefore$ B is (5, 2).

3 Suppose M is the midpoint of [AB]. For the points A and M below, find the coordinates of B using the midpoint formula. Check your answers using equal steps.

- **a** A is $(1, 3)$ and M is $(2, -1)$
- **b** A is $(2, 1)$ and M is $(0, 2)$
- **c** A is $(-2, 1)$ and M is $(-1\frac{1}{2}, 3)$
- **d** A is $(3, -2)$ and M is $(3\frac{1}{2}, -2)$
- **e** A is $(0, 0)$ and M is $(2, -\frac{1}{2})$
- **f** A is $(-3, \frac{1}{2})$ and M is $(0, 0)$.

4 P is the midpoint of [IJ]. Find the coordinates of I if:

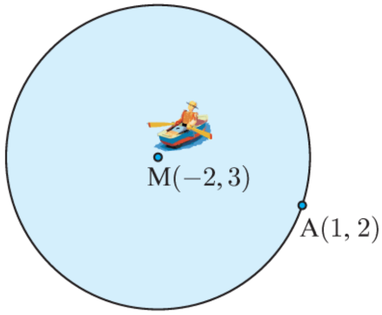
- **a** P is $(2, -6)$ and J is $(4, -3)$
- **b** P is $(0, -2)$ and J is $(-5, 1)$.

5 [PQ] is a diameter of a circle with centre C. If P is $(4, -7)$ and Q is $(-2, -3)$, find the coordinates of C.

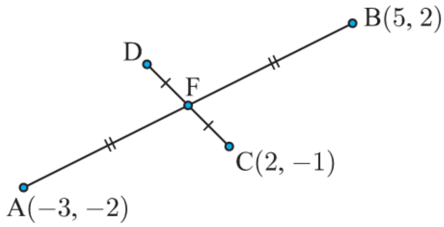
6 [AB] is a diameter of a circle with centre $(3\frac{1}{2}, -1)$. If B is $(2, 0)$, find the coordinates of A.

7 Torvald gets into a rowboat at $A(1, 2)$ on one side of a circular lake. He rows in a straight line across the lake, stopping in the middle for a rest at $M(-2, 3)$.

- **a** Find the coordinates of the point Torvald is aiming for.
- **b** If distances are in km, how much further does he have to row?



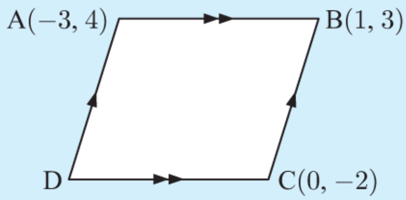
8



A flagpole at F is held by four wires pegged into the ground at A, B, C, and D. Opposite pegs are the same distance away from the pole. Find the coordinates of D.

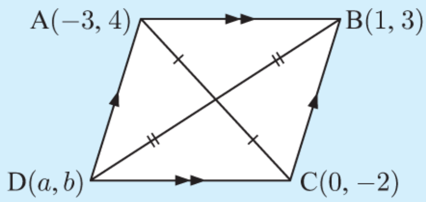
Example 5 **Self Tutor**

Use midpoints to find the fourth vertex of the given parallelogram:



Since ABCD is a parallelogram, the diagonals bisect each other.

$\therefore$ the midpoints of [DB] and [AC] coincide.



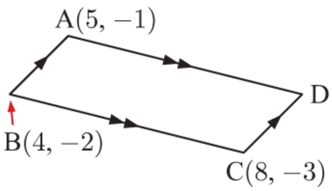
If D is (a, b), then $\dfrac{a+1}{2} = \dfrac{-3+0}{2}$ and $\dfrac{b+3}{2} = \dfrac{4+(-2)}{2}$

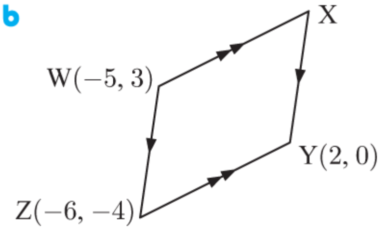
$\therefore\ a+1 = -3$ and $b+3 = 2$

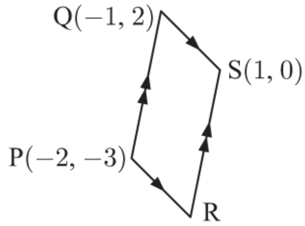
$\therefore\ a = -4$ and $b = -1$

So, D is $(-4, -1)$.

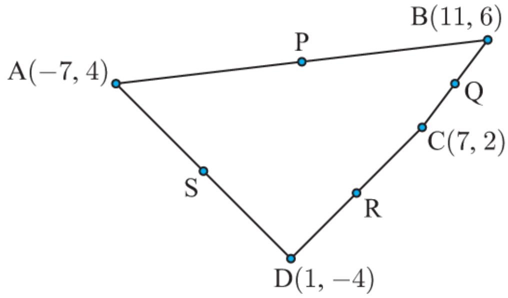
9 Use midpoints to find the fourth vertex of the parallelogram:

a


b


c


10 In the quadrilateral shown, P, Q, R, and S are the midpoints of [AB], [BC], [CD], and [DA] respectively.



a Find the coordinates of:
   i P **ii** Q **iii** R **iv** S

b Find the length of:
   i [PQ] **ii** [QR] **iii** [RS] **iv** [SP]

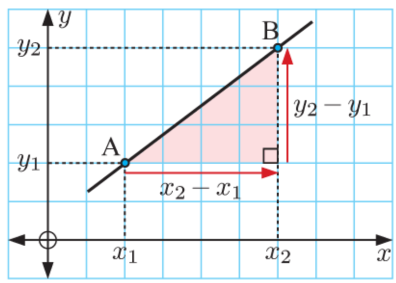
c What can be deduced about quadrilateral PQRS?

C GRADIENT

When looking at line segments drawn on a set of axes, it is clear that different line segments are inclined to the horizontal at different angles. Some appear to be *steeper* than others.

The **gradient** or **slope** of a line is a measure of its steepness.

If a line passes through $A(x_1, y_1)$ and $B(x_2, y_2)$, then the horizontal or x-step is $x_2 - x_1$, and the vertical or y-step is $y_2 - y_1$.



The **gradient** m of the line passing through (x_1, y_1) and (x_2, y_2) is $\boldsymbol{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$.

Example 6 **Self Tutor**

Find the gradient of the line through $(-2, 1)$ and $(2, 9)$.

$\underset{x_1\ \ y_1}{(-2,\ 1)}$ $\underset{x_2\ \ y_2}{(2,\ 9)}$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{9 - 1}{2 - (-2)}$

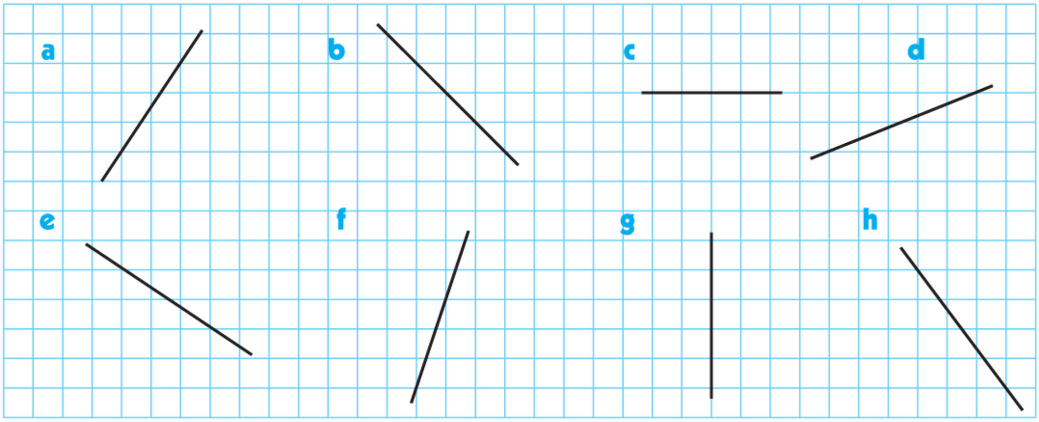
$= \frac{8}{4}$

$= 2$

The gradient of any horizontal line is 0 (zero).
The gradient of any vertical line is undefined.

EXERCISE 5C

1 Find the gradient of each line segment:



2 On grid paper, draw a line segment with gradient:

a $\frac{1}{4}$ **b** -2 **c** 4 **d** $-\frac{1}{4}$ **e** 0 **f** $\frac{4}{7}$

3 On the same set of axes, draw lines through $(3, 2)$ with gradients $\frac{1}{2}$, $\frac{2}{3}$, 3, and 5.

4 On the same set of axes, draw lines through $(2, -3)$ with gradients -4, $-\frac{5}{2}$, $-\frac{3}{4}$, $-\frac{1}{3}$, and 0.

5 Find the gradient of the line passing through:

a $(0, 0)$ and $(3, 5)$ **b** $(5, 2)$ and $(2, 9)$ **c** $(1, -4)$ and $(-5, -2)$
d $(-3, 4)$ and $(2, 4)$ **e** $(-6, 0)$ and $(0, -4)$ **f** $(-3, 5)$ and $(-3, 1)$

Example 7 **Self Tutor**

Find a given that the line through $P(a, -4)$ and $Q(1, 8)$ has gradient 3.

The gradient of [PQ] = 3, so $\dfrac{8-(-4)}{1-a} = 3$ {gradient formula}

$\therefore\ 12 = 3(1-a)$
$\therefore\ 12 = 3 - 3a$
$\therefore\ 3a = -9$
$\therefore\ a = -3$

6 Find a given that the line through:

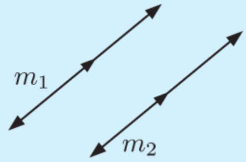
a $P(1, 5)$ and $Q(4, a)$ has gradient 2 **b** $M(-2, a)$ and $N(0, -2)$ has gradient -4
c $A(a, 8)$ and $B(-3, -4)$ has gradient $\frac{2}{3}$.

7 A line with gradient -2 passes through the point $(-1, 10)$. Determine where this line cuts the x-axis.

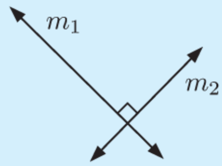
D PARALLEL AND PERPENDICULAR LINES

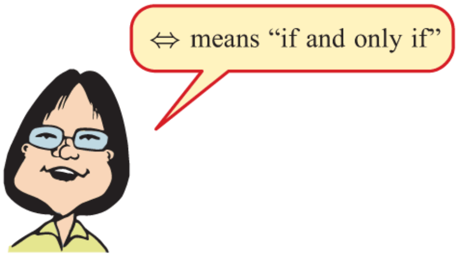
For non-vertical lines l_1 and l_2 with gradients m_1 and m_2 respectively:

- The lines are **parallel** if and only if they have the same gradient.
  l_1 is parallel to $l_2 \;\Leftrightarrow\; m_1 = m_2$.



- The lines are **perpendicular** if and only if their gradients are the *negative* reciprocals of each other.
  l_1 is perpendicular to $l_2 \;\Leftrightarrow\; m_2 = -\dfrac{1}{m_1}$





Example 8 **Self Tutor**

Find t given that the line joining A(1, 4) to B(5, t) is perpendicular to a line with gradient $\frac{2}{3}$.

The gradient of [AB] $= -\frac{3}{2}$ {perpendicular to line with gradient $\frac{2}{3}$}

$\therefore \; \dfrac{t-4}{5-1} = -\dfrac{3}{2}$ {gradient formula}

$\therefore \; \dfrac{t-4}{4} = \dfrac{-6}{4}$ {writing fractions with equal denominators}

$\therefore \; t - 4 = -6$ {equating numerators}

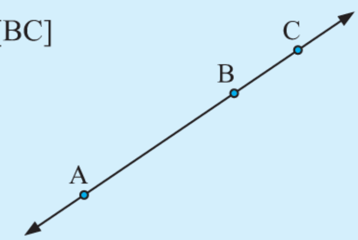
$\therefore \; t = -2$

COLLINEAR POINTS

Three or more points are **collinear** if they lie on the same straight line.

Three points A, B, and C are **collinear** if:

gradient of [AB] = gradient of [BC]



Example 9 **Self Tutor**

Show that the points A$(-5, -3)$, B$(-1, -1)$, and C$(7, 3)$ are collinear.

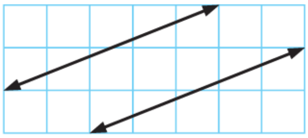
The gradient of [AB] $= \dfrac{-1-(-3)}{-1-(-5)}$
$= \frac{2}{4} = \frac{1}{2}$

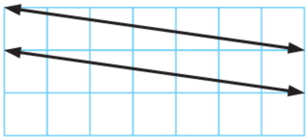
The gradient of [BC] $= \dfrac{3-(-1)}{7-(-1)}$
$= \frac{4}{8} = \frac{1}{2}$

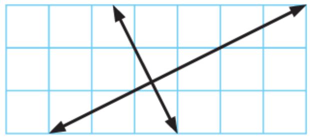
[AB] and [BC] have equal gradients, and so A, B, and C are collinear.

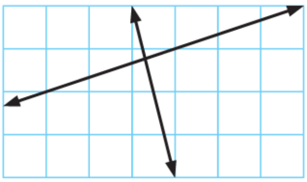
EXERCISE 5D

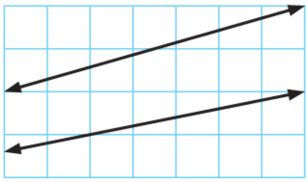
1 Classify the following pairs of lines as parallel, perpendicular, or neither. Give reasons for your answers.

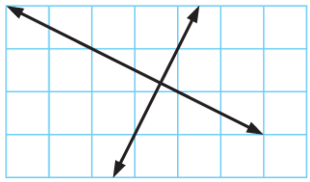
a


b


c


d


e


f


2 State the gradient of the line which is perpendicular to the line with gradient:

a $\frac{3}{4}$ **b** $\frac{11}{3}$ **c** 4 **d** $-\frac{1}{3}$ **e** -5 **f** 0

3 Given the gradients of two lines below, decide whether the lines are perpendicular:

a $2, \frac{1}{2}$ **b** $\frac{3}{5}, -\frac{3}{5}$ **c** $4, -\frac{1}{4}$ **d** $\frac{3}{4}, 1\frac{1}{3}$ **e** $-\frac{2}{5}, 2\frac{1}{2}$ **f** $-\frac{3}{8}, 2\frac{1}{3}$

4 Find t given that the line through:

a C$(2, 5)$ and D$(4, t)$ is perpendicular to a line with gradient $\frac{1}{4}$

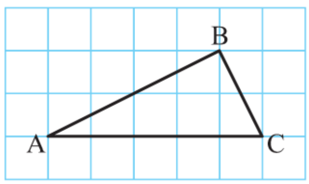
b X$(-3, -1)$ and Y$(t, 1)$ is perpendicular to a line with gradient $3\frac{1}{2}$.

5 Consider the points A$(1, 2)$, B$(-3, 0)$, C$(5, 3)$, and D$(3, k)$. Find k such that:

a [AB] is parallel to [CD]
b [AC] is parallel to [DB]
c [AB] is perpendicular to [CD]
d [AD] is perpendicular to [BC].

6 Show that triangle ABC is right angled using:

a Pythagoras' theorem
b the gradients of [AB] and [BC].



7 Determine whether these three points are collinear:

a A$(-1, 7)$, B$(1, 1)$, C$(4, -8)$
b P$(-4, 2)$, Q$(-1, 3)$, R$(5, 6)$
c R$(-2, 1)$, S$(4, 11)$, T$(-5, -4)$
d X$(7, -5)$, Y$(2, -1)$, Z$(-6, 5)$

8 Find n given that:

a A$(-7, -8)$, B$(-1, 1)$, and C$(3, n)$ are collinear
b P$(3, -11)$, Q$(n, -2)$, and R$(-5, 13)$ are collinear.

Background Knowledge 6

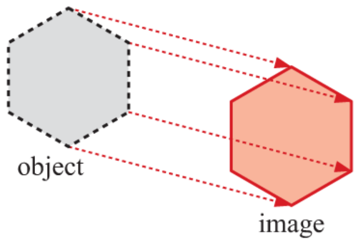
Transformation geometry

Contents:

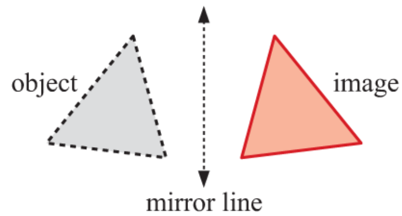
A **transformation** is a change in the position, orientation, or size of an object.
When a transformation is performed on an object, the result is called the **image**.

In this Chapter we study the following transformations:

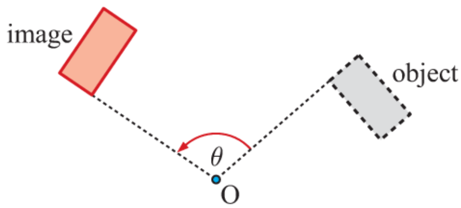
- **translations** where every point moves a fixed distance in a given direction



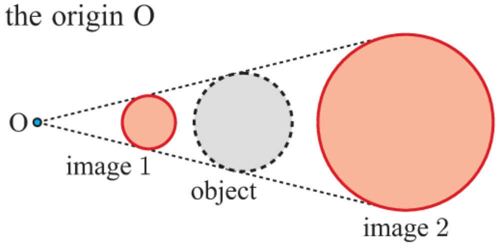
- **reflections** about mirror lines



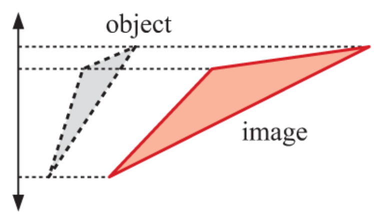
- **rotations** about a centre O through angle θ



- **dilations** which include:
  - **enlargements** and **reductions** about the origin O



  - **stretches** where one axis is fixed.



A TRANSLATIONS

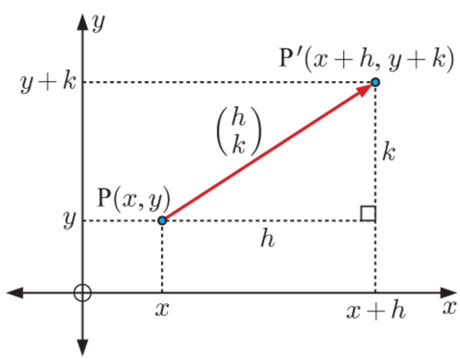
A **translation** moves a figure from one place to another. Every point on the figure moves the same distance in the same direction.

If $P(x, y)$ is **translated** h units in the x-direction and k units in the y-direction, then the image point P' has coordinates $(x+h,\ y+k)$.

We write $P(x, y) \xrightarrow{\binom{h}{k}} P'(x+h,\ y+k)$

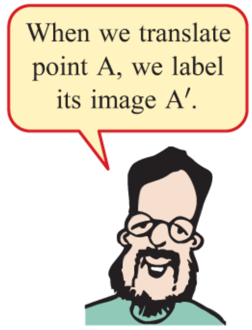
where P' is called the **image** of the object P, and

$\begin{pmatrix} h \\ k \end{pmatrix}$ is called the **translation vector**.



Example 1 **Self Tutor**

Rectangle ABCD has vertices A(1, 4), B(4, 4), C(4, 2), and D(1, 2). Find the image vertices when the rectangle is translated $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$. Illustrate the translation.

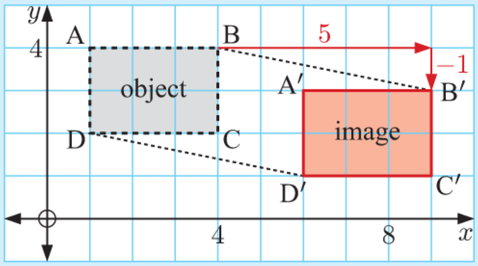


$A(1, 4) \xrightarrow{\binom{5}{-1}} A'(6, 3)$

$B(4, 4) \xrightarrow{\binom{5}{-1}} B'(9, 3)$

$C(4, 2) \xrightarrow{\binom{5}{-1}} C'(9, 1)$

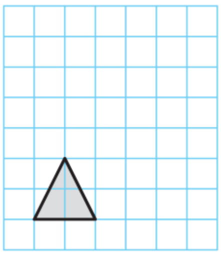
$D(1, 2) \xrightarrow{\binom{5}{-1}} D'(6, 1)$



EXERCISE 6A

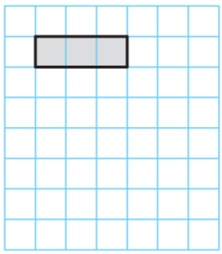
1 Transform each object in the direction given:

a



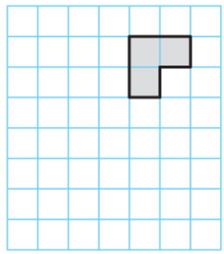
3 units right, 1 unit up

b



2 units right, 3 units down

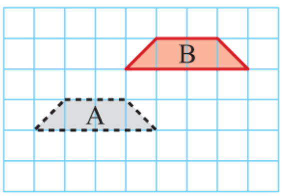
c



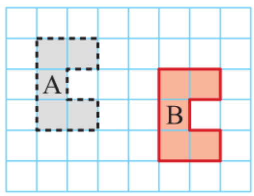
1 unit left, 4 units down

2 Find the translation vector for the following translations from A to B:

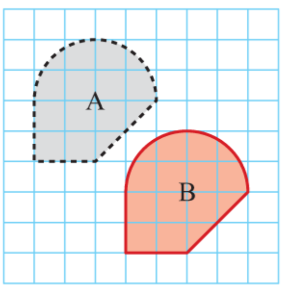
a



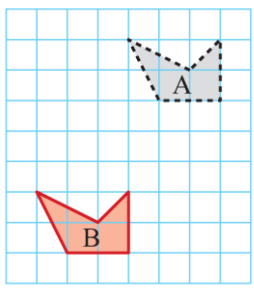
b



c



d



3 Find the image point when:

a $(-1, 3)$ is translated $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

b $(3, -2)$ is translated $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

4 Find the translation vector which translates the point:

a $(1, -4)$ to $(5, -1)$

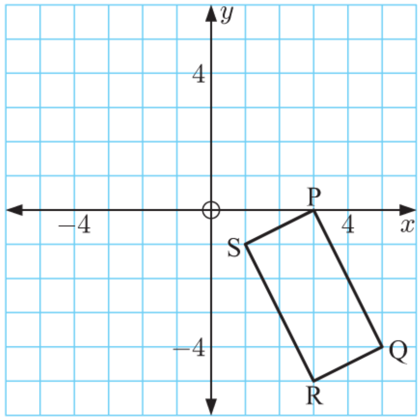
b $(3, 2)$ to $(-1, 4)$.

5 Find the point which has image:

a $(0, 3)$ under the translation $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

b $(4, -2)$ under the translation $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

6



PQRS is a rectangle.

a State the coordinates of P, Q, R, and S.

b Copy the rectangle, and translate it $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

c State the coordinates of the image vertices P′, Q′, R′, and S′.

7 Triangle ABC has vertices $A(3, 1)$, $B(-1, 2)$, and $C(0, 4)$.

a Draw triangle ABC on a set of axes.

b Translate the figure $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

c State the coordinates of the image vertices A′, B′, and C′.

d Find the distance each point has moved.

8 Find the single transformation equivalent to:

a a translation of $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ followed by a translation of $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

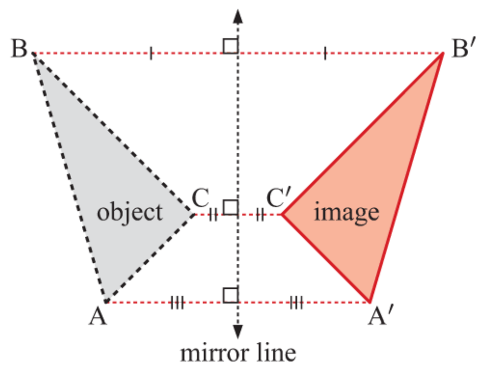
b a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ followed by a translation of $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

B REFLECTIONS

In the figure alongside, the triangle ABC has been **reflected** in the **mirror line** to form its image A′B′C′.

When an object is reflected in a mirror line:

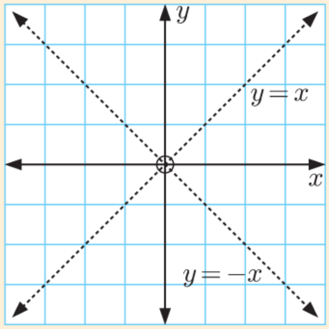
- All lines and angles are the same size in the image as they were in the object.
- The mirror line is the **perpendicular bisector** of the line segment joining a point on the object with its corresponding point on the image.



INVESTIGATION 1 — REFLECTIONS

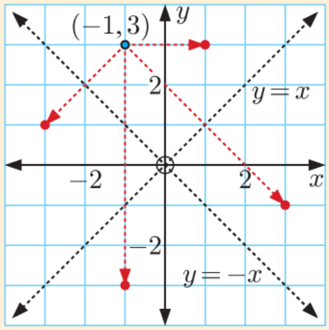
In this Investigation we consider reflections in four important lines:

- the x-axis
- the y-axis
- $y = x$
- $y = -x$



What to do:

1. The grid alongside shows reflections of $(-1, 3)$ in each of the four mirror lines. State the image of $(-1, 3)$ under a reflection in:
   - **a** the x-axis
   - **b** the y-axis
   - **c** $y = x$
   - **d** $y = -x$.



2. Draw your own grid, including the lines $y = x$ and $y = -x$. Draw the reflection, and state the image of $(2, 3)$ under a reflection in:
   - **a** the x-axis
   - **b** the y-axis
   - **c** $y = x$
   - **d** $y = -x$.
3. Study your answers to **1** and **2**. How do the coordinates of the object relate to the coordinates of the image under a reflection in:
   - **a** the x-axis
   - **b** the y-axis
   - **c** $y = x$
   - **d** $y = -x$?

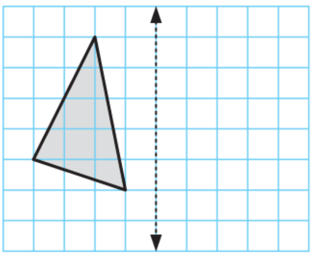
From the **Investigation** you should have observed these properties of reflecting a point (x, y):

Object	*Reflection*	*Image*
(x, y)	reflection in the x-axis	$(x, -y)$
(x, y)	reflection in the y-axis	$(-x, y)$
(x, y)	reflection in $y = x$	(y, x)
(x, y)	reflection in $y = -x$	$(-y, -x)$

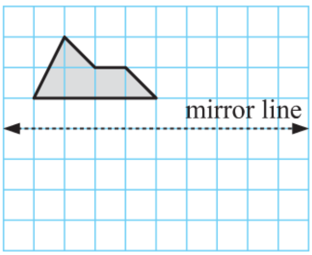
EXERCISE 6B

1. Reflect each object in the given mirror line:

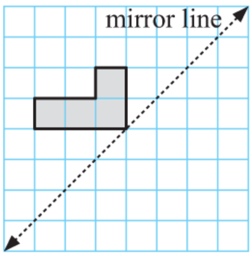
a



b



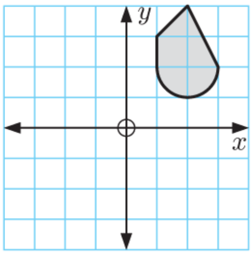
c



2 Reflect the given object in the:

a x-axis

b y-axis.



3 Find the image of $(4, -1)$ under a reflection in:

a the x-axis **b** the y-axis **c** $y = x$ **d** $y = -x$

Illustrate your answers.

4 Find the image of $(-3, -3)$ under a reflection in:

a the x-axis **b** the y-axis **c** $y = x$ **d** $y = -x$

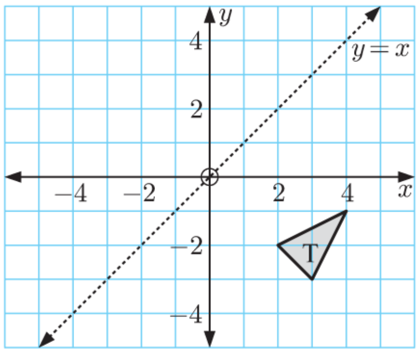
Illustrate your answers, and explain any unusual results.

5 Copy the graph given. Reflect T in:

a the x-axis and label it U

b the y-axis and label it V

c the line $y = x$ and label it W.



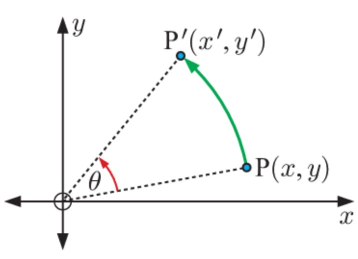
6 Find the image of:

a $(1, 3)$ under the translation $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ followed by a reflection in the x-axis

b $(2, 3)$ under a reflection in $y = -x$ followed by the translation $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

c $(5, 1)$ under a reflection in the y-axis followed by a reflection in $y = x$.

C ROTATIONS



If $P(x, y)$ is moved under a **rotation** about O through an angle of θ to a new position $P'(x', y')$, then $OP = OP'$ and $P\widehat{O}P' = \theta$.

θ is measured **positive** in the **anticlockwise** direction.

The centre of the rotation O is the only point which does not move under the rotation.

INVESTIGATION 2 ROTATIONS

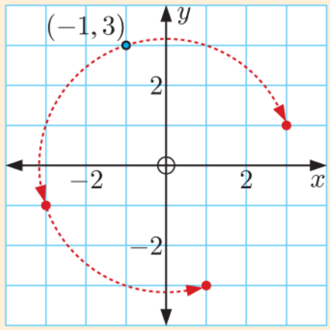
In this Investigation we consider rotations about the origin O of:

- 90° anticlockwise
- 180°
- 90° clockwise or -90°

What to do:

1 The grid alongside shows rotations of $(-1, 3)$ about the origin O. State the image of $(-1, 3)$ under the rotation:

a 90° anticlockwise **b** 180°
c 90° clockwise or -90°.



2 Plot the point $(2, 3)$ on a grid. Draw the rotation, and state the image of $(2, 3)$ under the rotation about O of:

a 90° anticlockwise **b** 180°
c 90° clockwise or -90°.

3 Study your answers to **1** and **2**. How do the coordinates of the object relate to the coordinates of the image under a rotation about O of:

a 90° anticlockwise **b** 180° **c** 90° clockwise or -90°?

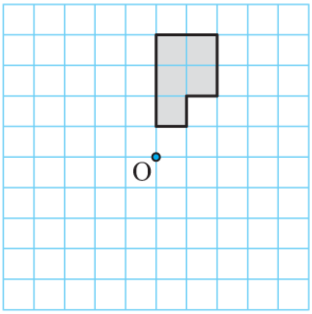
From the **Investigation** you should have observed these properties of rotating a point (x, y) about O:

Object	*Rotation*	*Image*
(x, y)	90° anticlockwise	$(-y, x)$
(x, y)	180°	$(-x, -y)$
(x, y)	90° clockwise or -90°	$(y, -x)$

EXERCISE 6C

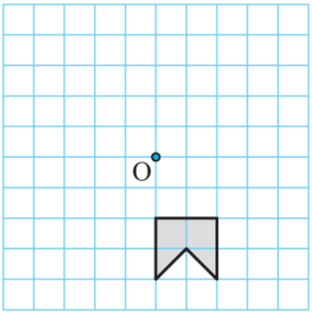
1 Rotate each figure about O through the angle given:

a



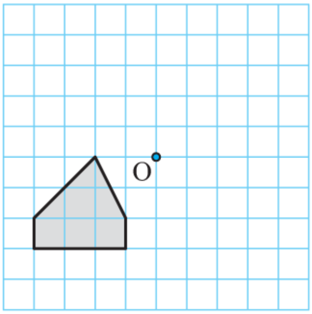
90° anticlockwise

b



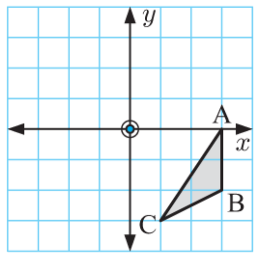
180°

c



90° clockwise or -90°

2 **a** Rotate triangle ABC $90°$ clockwise about the origin O.
b State the vertex coordinates of the image.



3 Find the image of the point $(-2, 3)$ under a rotation about O of:
a $90°$ anticlockwise **b** $90°$ clockwise **c** $180°$

4 Find the image of the point $(4, -1)$ under a rotation about O of:
a $90°$ anticlockwise **b** $90°$ clockwise **c** $180°$

5 Triangle ABC has vertices $A(1, 4)$, $B(-1, 5)$, and $C(-2, 2)$. It is rotated anticlockwise through $90°$ about O.
a Draw triangle ABC and its image $A'B'C'$.
b State the coordinates of A', B', and C'.

6 Triangle PQR with $P(2, -4)$, $Q(3, -2)$, and $R(1, -1)$ is rotated about O through $180°$.
a Draw triangle PQR and its image $P'Q'R'$.
b State the coordinates of P', Q', and R'.

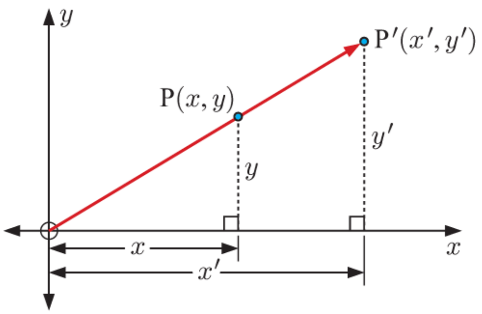
7 Find the image of:
a $(4, 1)$ under a reflection in the y-axis followed by a $90°$ clockwise rotation
b $(3, -2)$ under a $180°$ rotation followed by a translation through $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$
c $(-3, -1)$ under a reflection in $y = x$ followed by a $90°$ anticlockwise rotation.

D DILATIONS (ENLARGEMENTS, REDUCTIONS, AND STRETCHES)

ENLARGEMENTS AND REDUCTIONS

Suppose $P(x, y)$ moves to $P'(x', y')$ such that P' lies on the line (OP), and $OP' = k OP$.

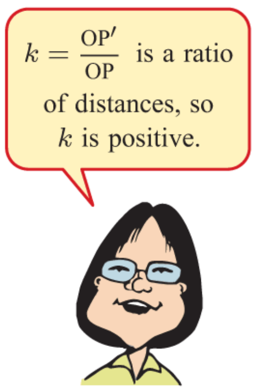
We call this a **dilation** with centre O and **scale factor** k.



From the similar triangles,

$$\frac{x'}{x} = \frac{y'}{y} = \frac{OP'}{OP} = k$$

$$\therefore \quad x' = kx \quad \text{and} \quad y' = ky$$



For a **dilation** with **centre O**$(0,\ 0)$ and **scale factor** k, $(x,\ y)$ moves to $(kx,\ ky)$.

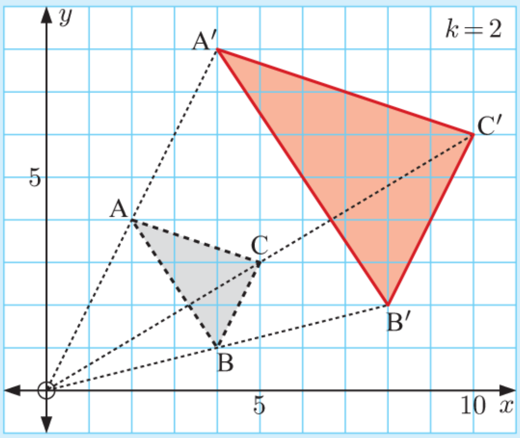
If $k > 1$, the image is an **enlargement** of the object.

If $0 < k < 1$, the image is a **reduction** of the object.

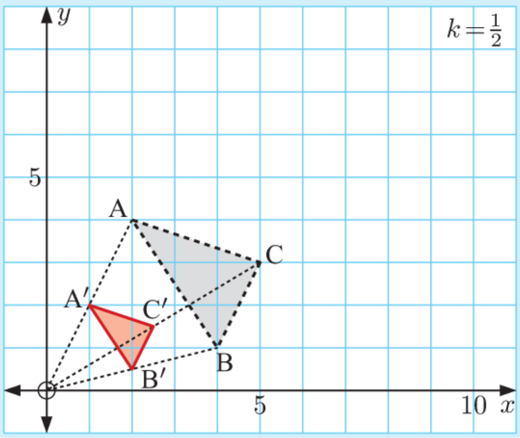
Example 2 **Self Tutor**

Consider the triangle ABC with vertices A$(2,\ 4)$, B$(4,\ 1)$, and C$(5,\ 3)$. Draw the image of triangle ABC under a dilation with centre O and scale factor: **a** $k = 2$ **b** $k = \frac{1}{2}$.

a



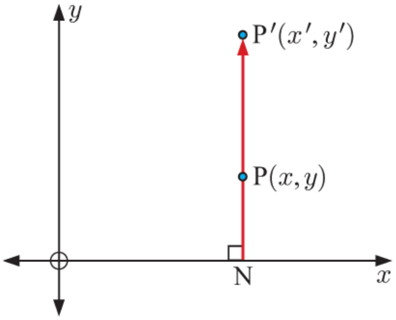
b



DISCUSSION — NEGATIVE SCALE FACTORS

- Do we need to consider negative scale factors?
- What combination of transformations would be equivalent to a dilation with centre O and scale factor $k = -2$?
- If we allowed negative scale factors, what would this say about the *uniqueness* of describing a series of transformations?

VERTICAL STRETCHES WITH FIXED x-AXIS



For a vertical stretch with fixed x-axis, we stretch the figure in the vertical direction only.

Suppose P$(x,\ y)$ moves to P$'(x',\ y')$ such that P$'$ lies on the line through N$(x,\ 0)$ and P, and NP$' = k$NP.

We call this a **vertical stretch** or **vertical dilation** with scale factor k.

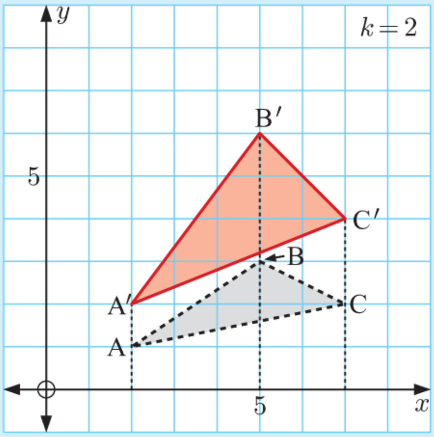
For a **vertical stretch** with **scale factor** k, the point $(x,\ y)$ moves to $(x,\ ky)$.

Example 3 **Self Tutor**

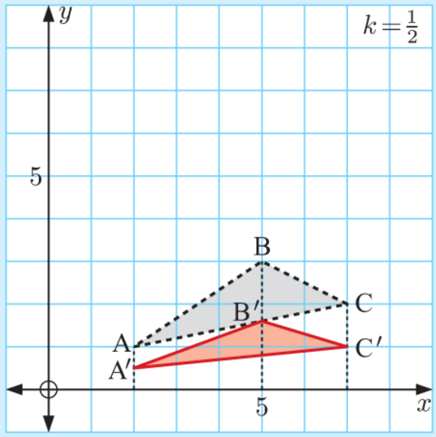
Consider the triangle ABC with A(2, 1), B(5, 3), and C(7, 2). Draw the image of triangle ABC under a vertical stretch with scale factor:

a $k = 2$ **b** $k = \frac{1}{2}$.

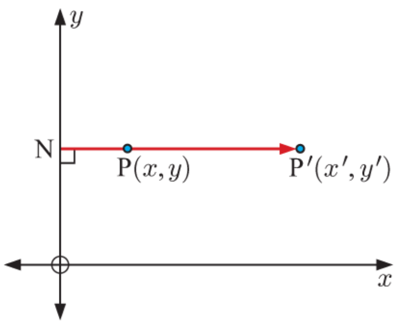
a



b



HORIZONTAL STRETCHES WITH FIXED y-AXIS



For a horizontal stretch with fixed y-axis, we stretch the figure in the horizontal direction only.

Suppose P(x, y) moves to P′(x', y') such that P′ lies on the line through N$(0, y)$ and P, and NP′ $= k$NP.

We call this a **horizontal stretch** or **horizontal dilation** with scale factor k.

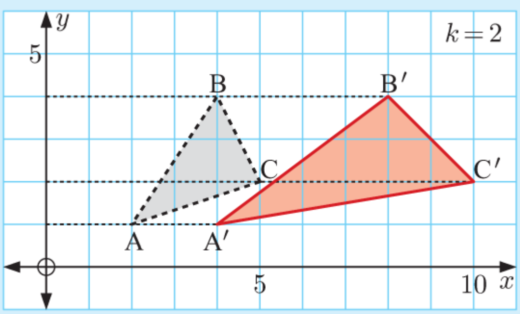
For a **horizontal stretch** with **scale factor** k, the point (x, y) moves to (kx, y).

Example 4 **Self Tutor**

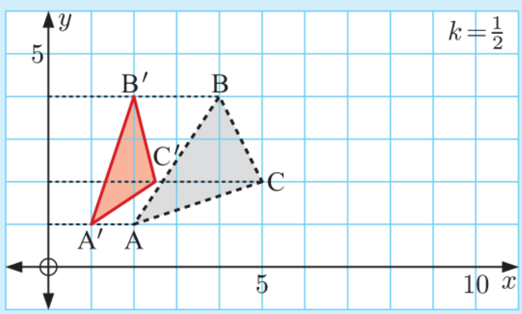
Consider the triangle ABC with A(2, 1), B(4, 4), and C(5, 2). Draw the image of triangle ABC under a horizontal stretch with scale factor:

a $k = 2$ **b** $k = \frac{1}{2}$.

a

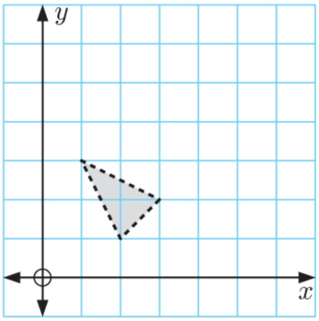


b

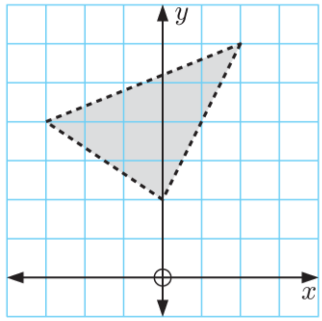


EXERCISE 6D

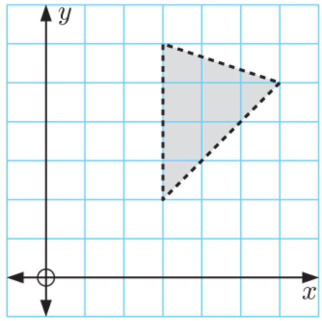
1 Copy each figure onto grid paper and perform the given dilation:

a


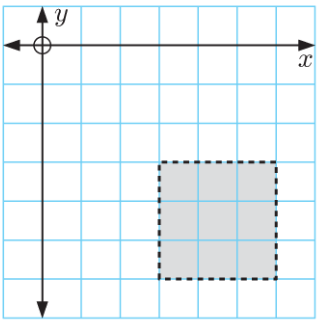
centre O
scale factor $k = 2$

b


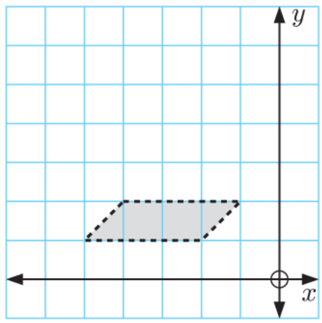
fixed x-axis
scale factor $k = \frac{1}{2}$

c


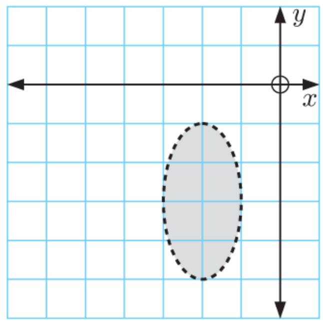
fixed y-axis
scale factor $k = \frac{1}{3}$

d


centre O
scale factor $k = \frac{1}{3}$

e


fixed x-axis
scale factor $k = 3$

f


fixed y-axis
scale factor $k = 2$

PRINTABLE DIAGRAMS

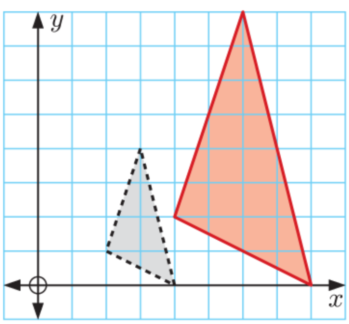
2 Find the image of:

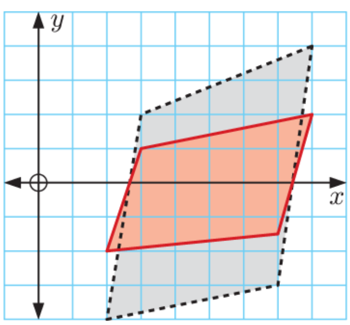
- **a** $(3, -1)$ under an enlargement with centre O and scale factor 2
- **b** $(-3, 6)$ under a reduction with centre O and scale factor $\frac{2}{3}$
- **c** $(-2, -3)$ under a vertical stretch with scale factor 3
- **d** $(1, 6)$ under a vertical stretch with scale factor $\frac{1}{2}$
- **e** $(-4, 3)$ under a horizontal stretch with scale factor $\frac{3}{4}$
- **f** $(1.2, 2)$ under a horizontal stretch with scale factor 5.

3 Sketch the image of a circle with centre O and radius 3 units under:

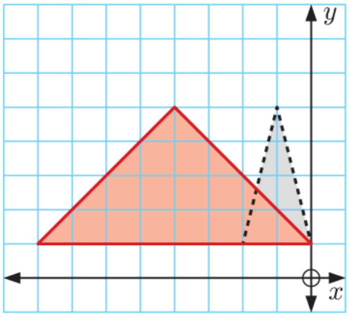
- **a** a dilation with centre O and scale factor 2
- **b** a vertical stretch with scale factor $\frac{2}{3}$
- **c** a horizontal stretch with scale factor $\frac{1}{3}$.

4 Describe the transformation which maps the object onto the image:

a


b


c



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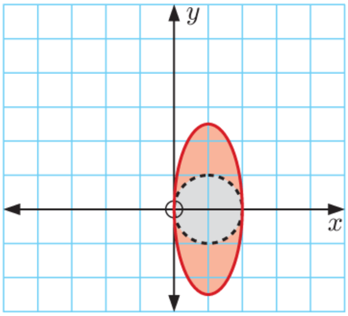

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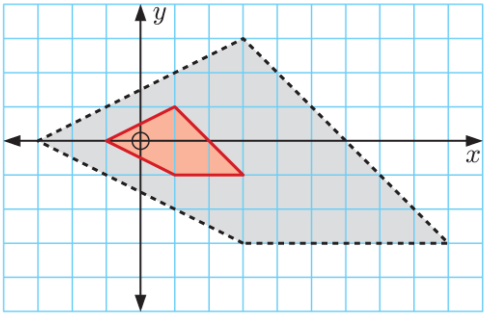

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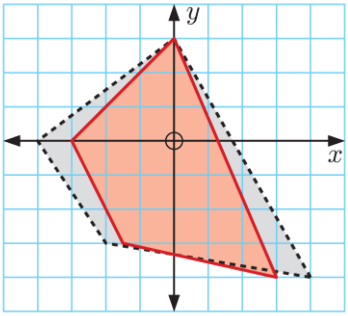


Background Knowledge

7

Similarity

Contents:

A SIMILAR FIGURES

Two figures are **similar** if one is an enlargement of the other.

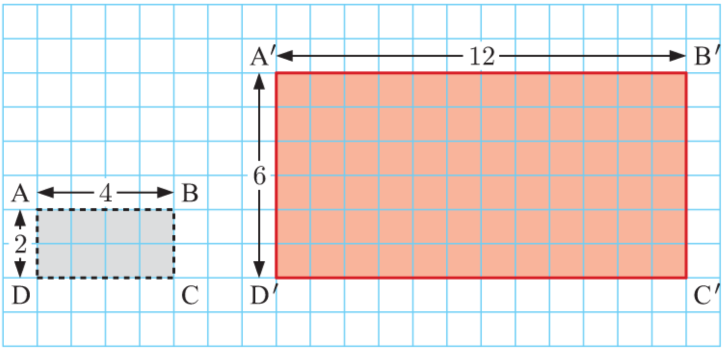
A′B′C′D′ is an enlargement of ABCD. The two figures are therefore similar.

Notice that

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{D'A'}{DA} = 3,$$

so the corresponding side lengths are in the **same ratio**.

We say that $k = 3$ is the **scale factor** for the enlargement.



When a figure is enlarged or reduced, the sizes of its angles do not change. The figures are therefore **equiangular**.

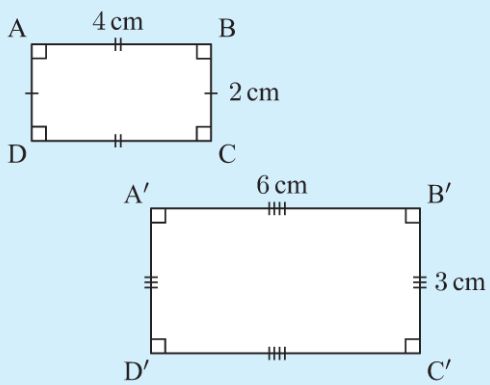
Two figures are **similar** if:
- the figures are **equiangular** *and*
- the corresponding side lengths are in the **same ratio**.

We say that the figures are *in proportion*.

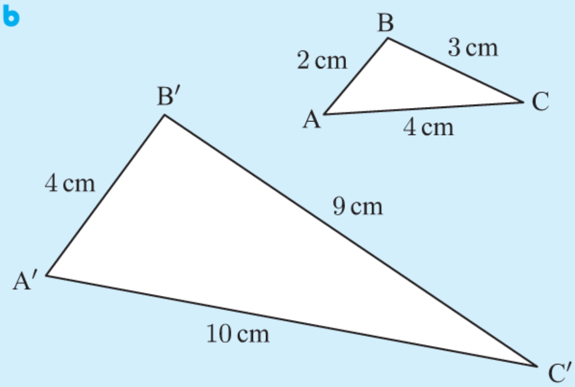
Example 1 **Self Tutor**

Determine whether the following pairs of figures are similar:

a



b



a $\frac{A'B'}{AB} = \frac{6}{4} = \frac{3}{2}$ and $\frac{B'C'}{BC} = \frac{3}{2}$

$\therefore$ the corresponding side lengths are in the same ratio.

The figures are also equiangular, so the figures are similar.

b $\frac{A'B'}{AB} = \frac{4}{2} = 2$ and $\frac{B'C'}{BC} = \frac{9}{3} = 3$

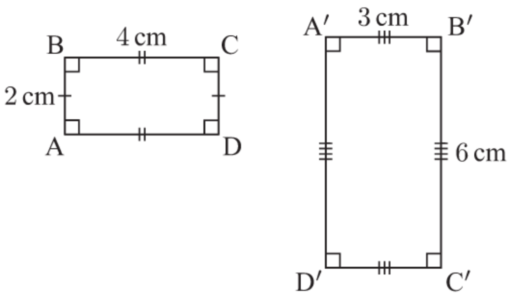
$\therefore$ the corresponding side lengths are *not* in the same ratio.

$\therefore$ the figures are not similar.

EXERCISE 7A

1 Determine whether the following pairs of figures are similar:

a



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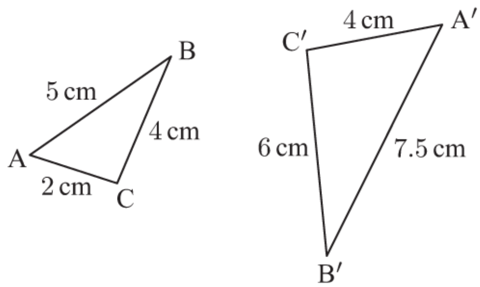

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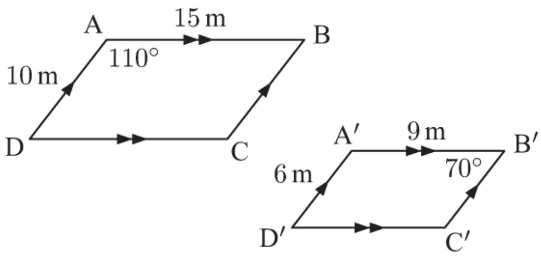

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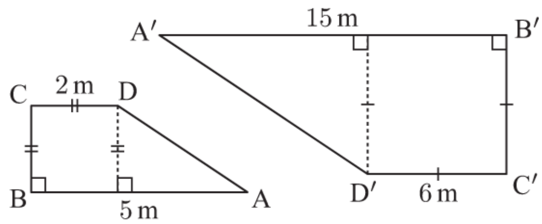
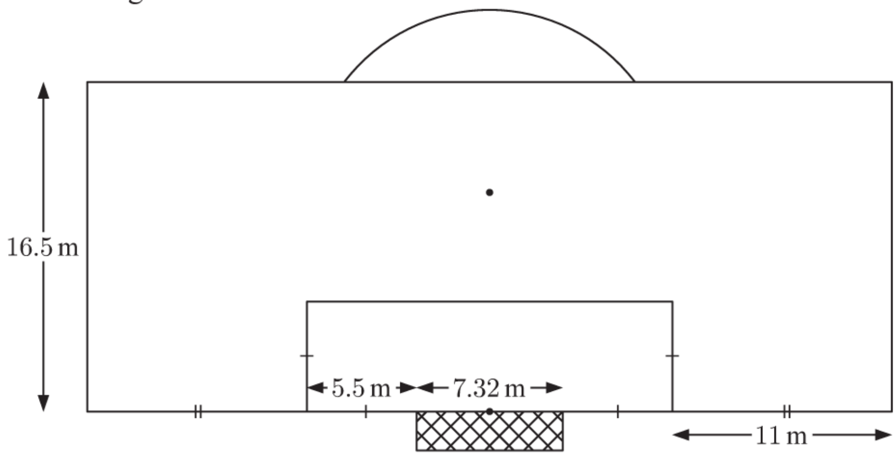


2 On a standard football pitch, the penalty box and the goal area have the dimensions shown. Are the two rectangles similar?



3 Comment on the truth of the following statements. If a statement is false, justify your answer with an illustration.

a All squares are similar.

b All rectangles are similar.

c All semi-circles are similar.

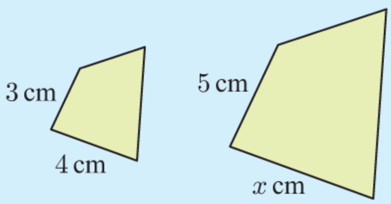
d All right-angled triangles are similar.

e All spheres are similar.

f All cones are similar.

Example 2 — **Self Tutor**

These figures are similar.
Find x, rounded to 2 decimal places.



Since the figures are similar, their corresponding sides are in the same ratio.

$$\therefore \quad \frac{x}{4} = \frac{5}{3}$$

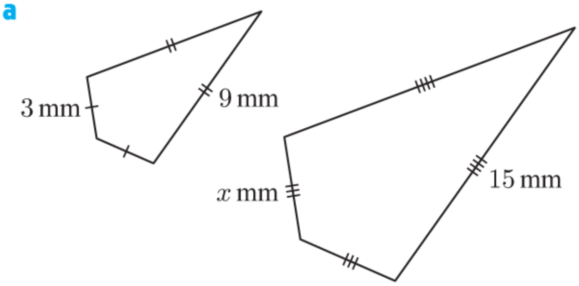
$$\therefore \quad x = \tfrac{5}{3} \times 4$$

$$\therefore \quad x = \tfrac{20}{3}$$

$$\therefore \quad x \approx 6.67$$

4 Given that the figures are similar, find the exact value of x:

a



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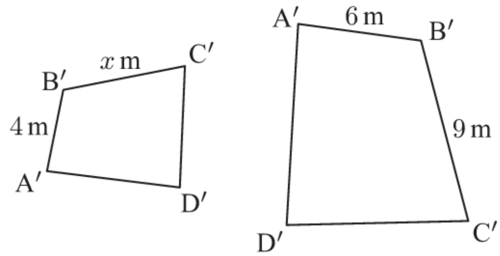

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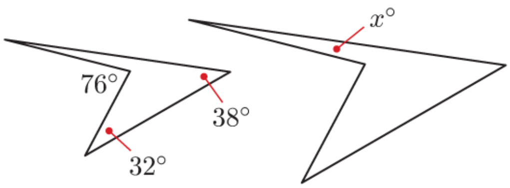

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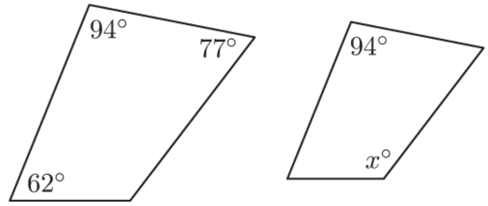
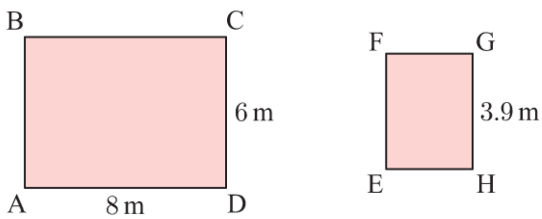
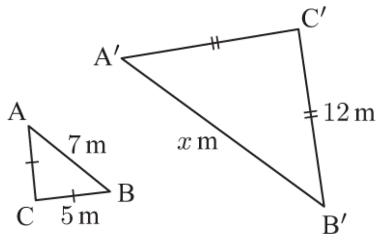


5 Rectangles ABCD and FGHE are similar. Find the length of FG.



6 Find x given that triangle ABC is similar to triangle A′B′C′:

a



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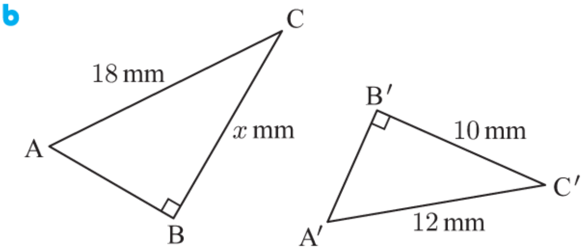

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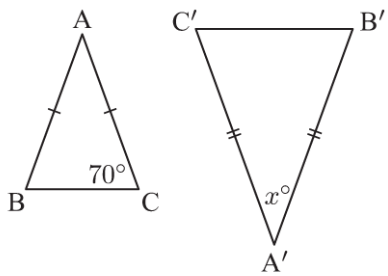
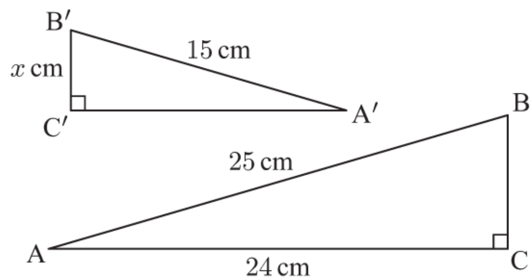


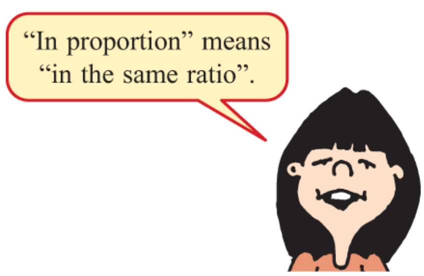
d



7 Sketch two quadrilaterals that:

a are equiangular, but not similar

b have sides in proportion, but are not similar.



B SIMILAR TRIANGLES

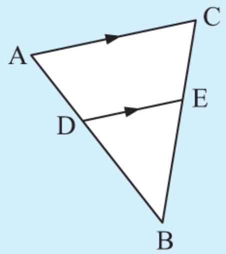
Two triangles are similar if either:
- they are equiangular *or* • their side lengths are in the same ratio.

Notice that:
- either of these properties is sufficient to prove that two triangles are similar
- since the angles of any triangle add up to $180°$, if two angles of one triangle are equal to two angles of another triangle, then the remaining angles of the triangles must also be equal.

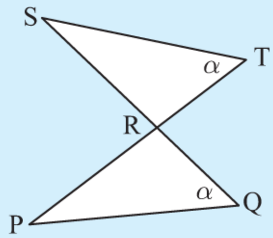
Example 3 **Self Tutor**

Show that the following figures possess similar triangles:

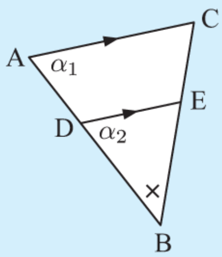
a



b



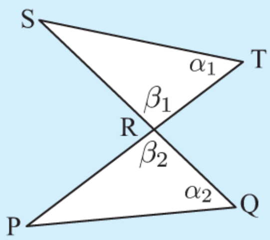
a



$\triangle$s ABC and DBE are equiangular as:
- $\alpha_1 = \alpha_2$ {equal corresponding angles}
- The angle at B is common to both triangles.

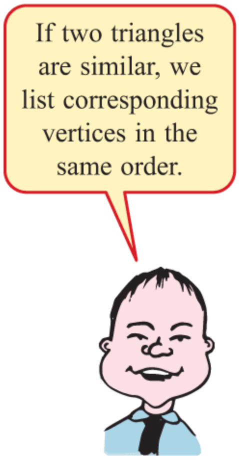
$\therefore$ the triangles are similar.

b



$\triangle$s PQR and STR are equiangular as:
- $\alpha_1 = \alpha_2$ {given}
- $\beta_1 = \beta_2$ {vertically opposite angles}

$\therefore$ the triangles are similar.

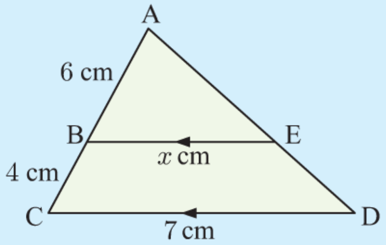


FINDING SIDE LENGTHS

Once we have established that two triangles are similar, we may use the fact that corresponding sides are in the same ratio to find unknown lengths.

Example 4 — Self Tutor

Establish that a pair of triangles is similar, and find x:



$\triangle$s ABE and ACD are equiangular as:

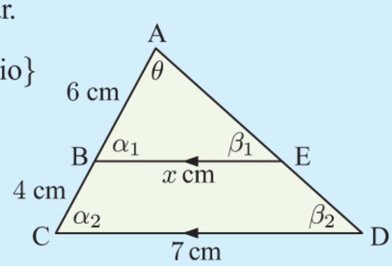
- $\alpha_1 = \alpha_2$ {equal corresponding angles}
- $\beta_1 = \beta_2$ {equal corresponding angles}

$\therefore$ the triangles are similar.

$\therefore \dfrac{BE}{CD} = \dfrac{AB}{AC}$ {same ratio}

$\therefore \dfrac{x}{7} = \dfrac{6}{6+4}$

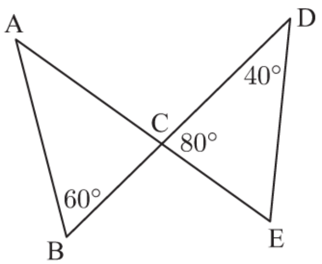
$\therefore x = \dfrac{6}{10} \times 7 = 4.2$



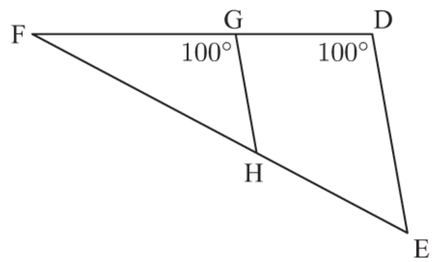
EXERCISE 7B

1 Show that the following figures possess similar triangles:

a



b



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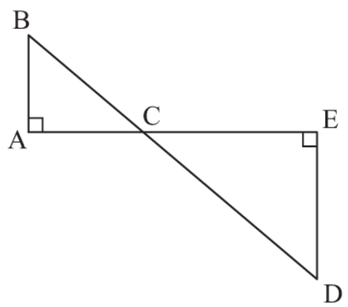

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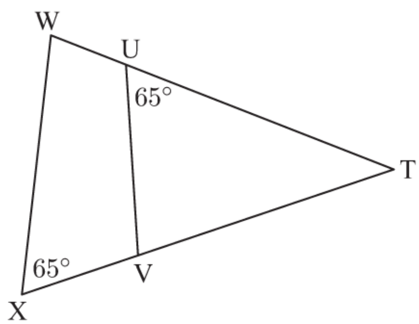
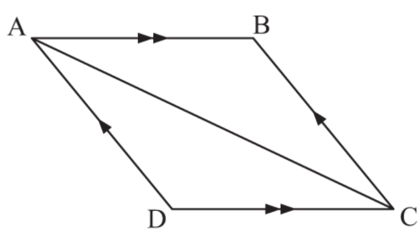


e



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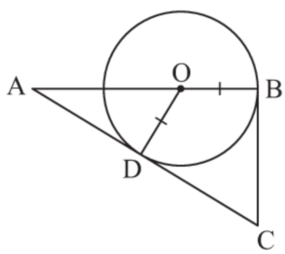
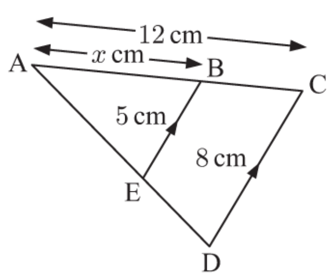


2 In each figure, establish that a pair of triangles is similar. Hence find x.

a



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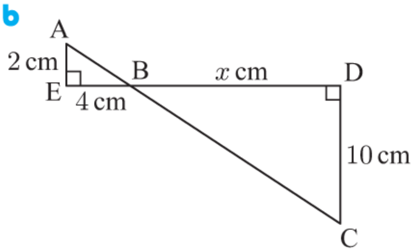

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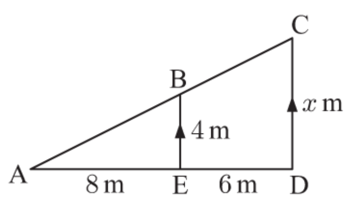

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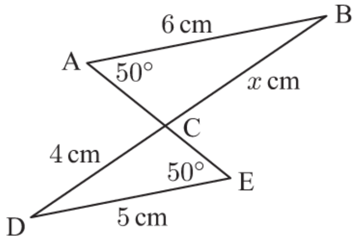

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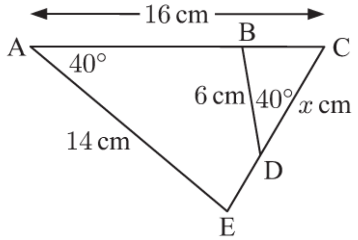

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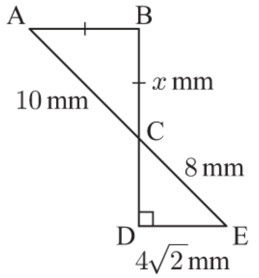
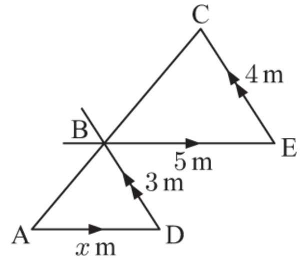


g



C PROBLEM SOLVING

The properties of similar triangles have been known since ancient times. However, even with the technologically advanced measuring instruments available today, similar triangles are still important for finding heights and distances which would otherwise be difficult to measure.

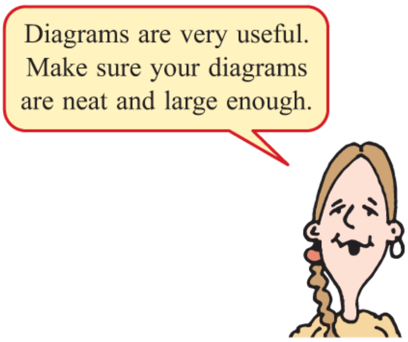
Step 1: Read the question carefully. Draw a diagram showing all of the given information.

Step 2: Introduce a variable such as x, for the unknown quantity to be found.

Step 3: Establish that a pair of triangles are similar, and hence write an equation involving the variable.

Step 4: Solve the equation.

Step 5: Answer the question in a sentence.



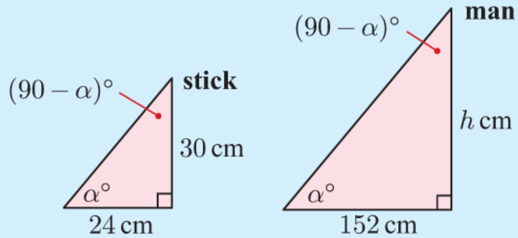
Example 5 **Self Tutor**

When a 30 cm stick is stood vertically on the ground, it casts a 24 cm shadow.
At the same time a man casts a shadow of length 152 cm. How tall is the man?

The sun shines at the same angle on both the stick and the man.
We suppose this is angle $\alpha°$ to the horizontal.

Let the man be h cm tall.

The triangles are equiangular and therefore similar.



$\therefore \quad \dfrac{h}{30} = \dfrac{152}{24}$ {same ratio}

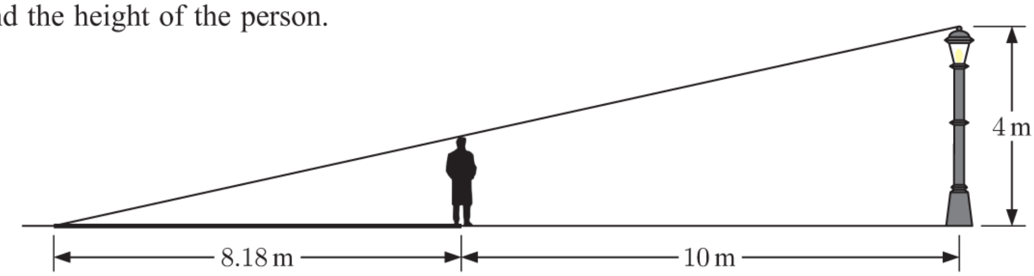
$\therefore \quad h = \dfrac{152}{24} \times 30$

$\therefore \quad h = 190$

The man is 190 cm tall.

EXERCISE 7C

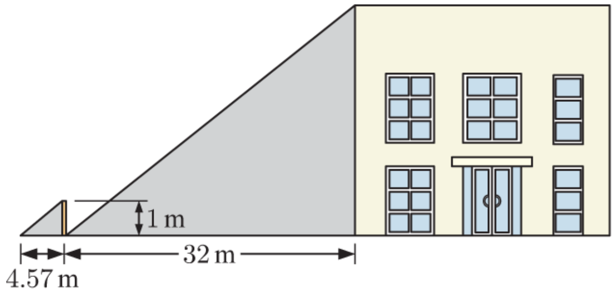
1 Find the height of the person.



2 Stevie wants to measure the height of a building. She stands at the edge of the building's shadow with a 1 m ruler.

  a Find the height of the building.

  b Does Stevie necessarily have to stand at the edge of the shadow? Explain your answer.



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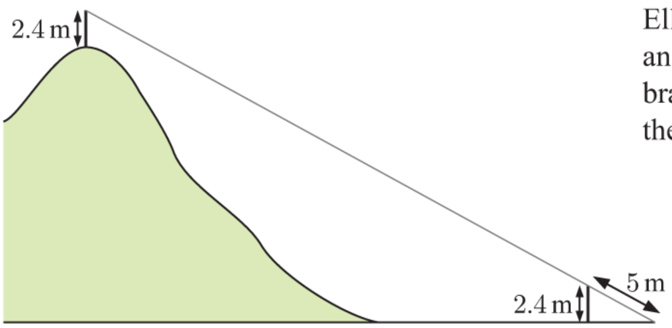
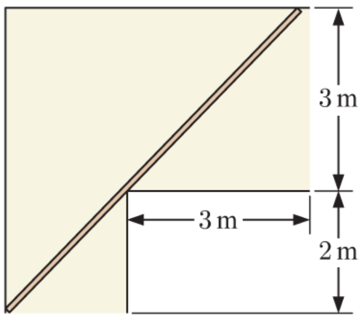
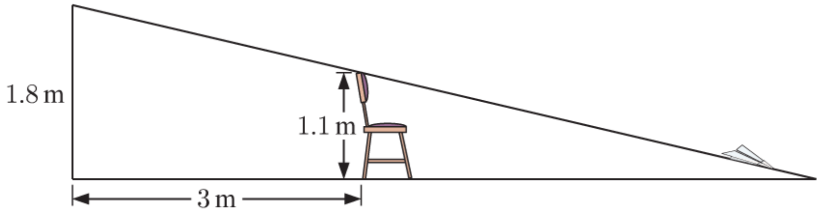


Ella is constructing a zip-line from the top of an 18 m high hill as shown. She will set up a braking system for the last 5 m of cable. Find the total length of the cable.

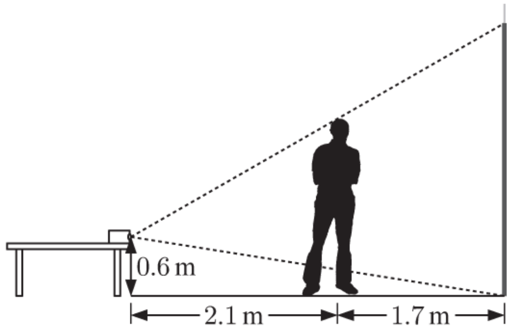
4 Harjeet's ladder has gotten stuck around a corner. How long is the ladder?



5 A paper plane is released from a height of 1.8 m. It passes over a chair and continues to the ground. How far does the paper plane fly through the air?



6 A 1.8 m tall man stands in front of a projector as shown. How far does his shadow extend up the wall?



D AREAS AND VOLUMES OF SIMILAR OBJECTS

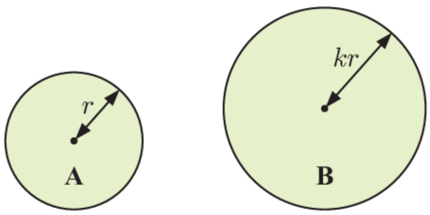
We have seen that if two objects are similar, then there is a relationship between the lengths of their sides. The sides are *in proportion*.

For 2 and 3 dimensional objects, we can establish relationships between the areas and the volumes of the objects.

For example:

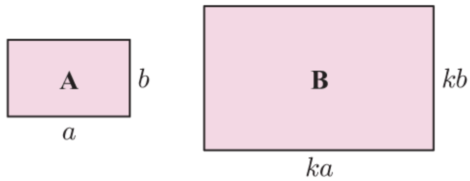
- The two circles shown are similar. Circle **B** is an enlargement of circle **A** with scale factor k.

  Area of **B** $= \pi(kr)^2$
  $= k^2 \times \pi r^2$
  $= k^2 \times$ area of **A**

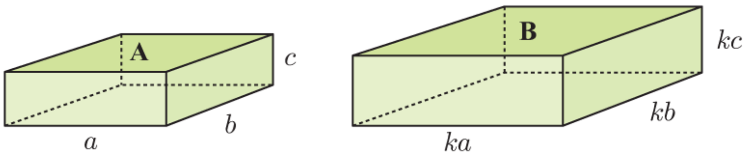


- The two rectangles shown are also similar. Rectangle **B** is an enlargement of rectangle **A** with scale factor k.

  Area of **B** $= ka \times kb$
  $= k^2 \times ab$
  $= k^2 \times$ area of **A**



- The two boxes shown are similar. Box **B** is an enlargement of box **A** with scale factor k.

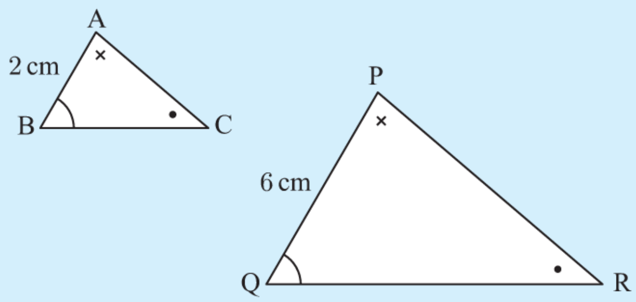


Volume of **B**
$= ka \times kb \times kc$
$= k^3 \times abc$
$= k^3 \times$ volume of **A**

Using examples like this we can conclude that:

- If a 2-dimensional object is enlarged with scale factor k to produce a similar object, the new area $= k^2 \times$ the old area.
- If a 3-dimensional object is enlarged with scale factor k to produce a similar object, the new volume $= k^3 \times$ the old volume.

Example 6 **Self Tutor**



Triangles ABC and PQR are equiangular and therefore similar.

AB = 2 cm and PQ = 6 cm.

The area of $\triangle$ABC is 5 cm^2.

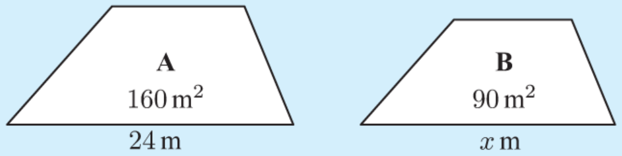
Find the area of $\triangle$PQR.

Suppose we enlarge $\triangle$ABC with scale factor k to give $\triangle$PQR.

$\therefore\ k = \frac{6}{2} = 3$

$$\begin{aligned}\text{Area } \triangle\text{PQR} &= k^2 \times \text{area } \triangle\text{ABC} \\ &= 9 \times 5 \text{ cm}^2 \\ &= 45 \text{ cm}^2\end{aligned}$$

Example 7 **Self Tutor**



Given that figures **A** and **B** are similar, find x.

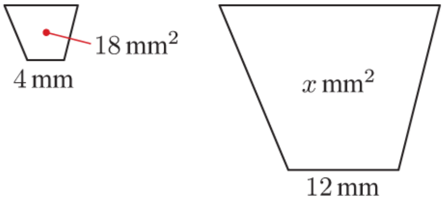
Figure **A** is reduced with scale factor k to give figure **B**.

$$\begin{aligned}\text{Area of } \mathbf{B} &= k^2 \times \text{area of } \mathbf{A} \\ \therefore\ 90 &= k^2 \times 160 \\ \therefore\ \tfrac{9}{16} &= k^2 \\ \therefore\ k &= \tfrac{3}{4} \qquad \{k > 0\} \\ \text{Now}\quad x &= \tfrac{3}{4} \times 24 \qquad \{\text{sides in the same ratio}\} \\ \therefore\ x &= 18\end{aligned}$$

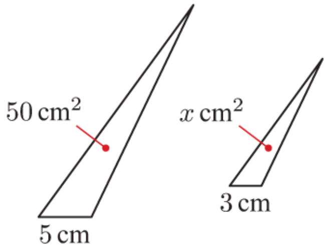
EXERCISE 7D

1 For each pair of similar shapes, find the unknown area:

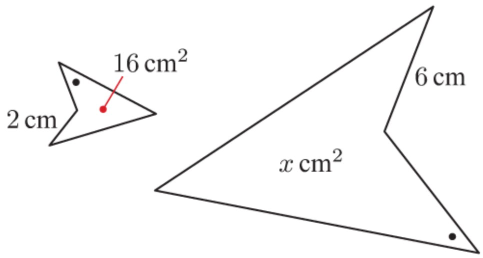
a



b

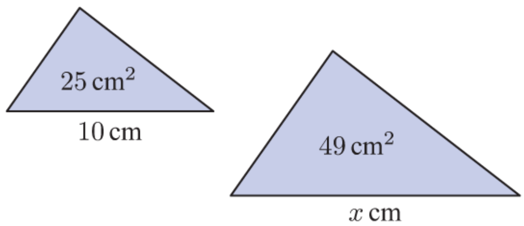


c

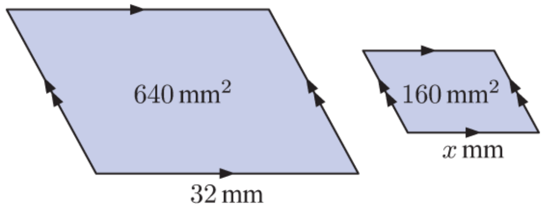


2 For each pair of similar shapes, find the unknown length:

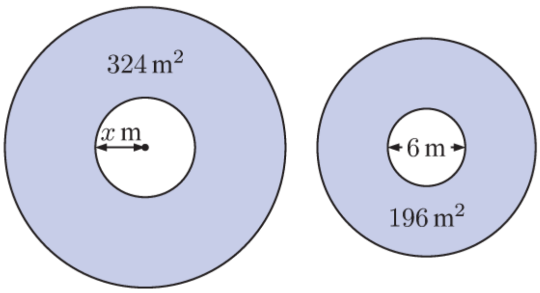
a



b



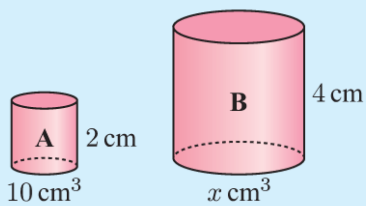
c



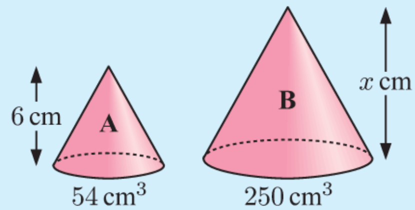
Example 8 **Self Tutor**

Given the following similar objects, find the unknown value:

a



b



a Suppose cylinder **A** is enlarged with scale factor k to give cylinder **B**.

$\therefore \quad k = \frac{4}{2} = 2$

Volume of **B** $= k^3 \times$ volume of **A**

$= 8 \times 10 \text{ cm}^3$

$= 80 \text{ cm}^3$

$\therefore \quad x = 80$

b Suppose cone **A** is enlarged with scale factor k to give cone **B**.

Volume of **B** $= k^3 \times$ volume of **A**

$\therefore \quad 250 = k^3 \times 54$

$\therefore \quad \frac{125}{27} = k^3$

$\therefore \quad k = \frac{5}{3}$

So, the height of **B** $= \frac{5}{3} \times$ the height of **A**

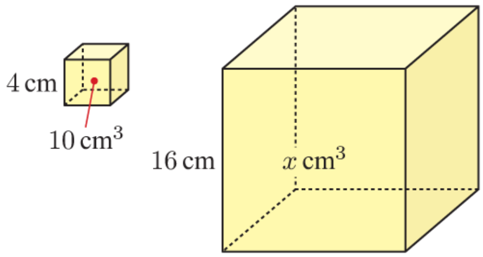
$= \frac{5}{3} \times 6 \text{ cm}$

$= 10 \text{ cm}$

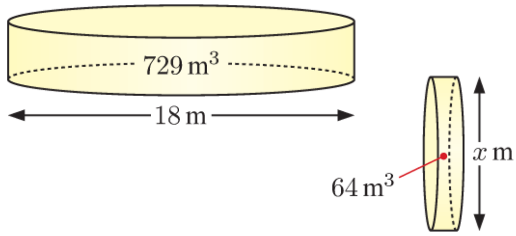
$\therefore \quad x = 10$

3 Given the following similar objects, find the unknown value:

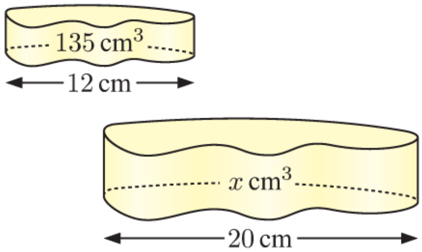
a



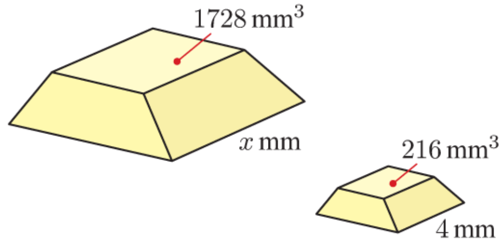
b



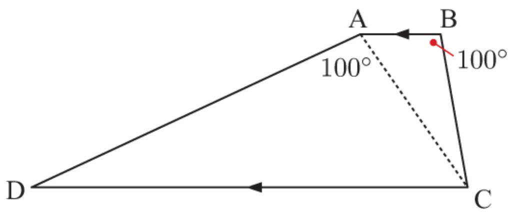
c



d



4



In the given figure, AB = 9 cm, AC = 21 cm, and $\triangle$ABC has area 39.9 cm^2.

a Show that $\triangle$ABC and $\triangle$CAD are similar.

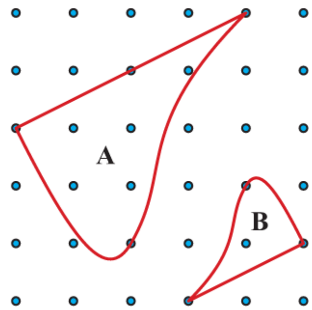
b Find the scale factor to enlarge $\triangle$ABC into $\triangle$CAD.

c Find the area of:

i $\triangle$CAD **ii** trapezium ABCD.

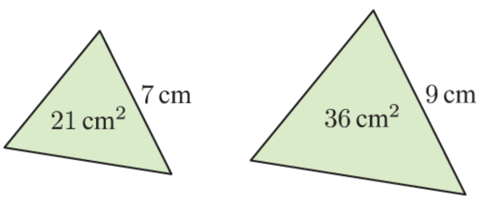
5 Figures **A** and **B** are similar. The dots shown are equally spaced at intervals of 1 unit.

If figure **A** has area 6.4 units^2, find the area of figure **B**.

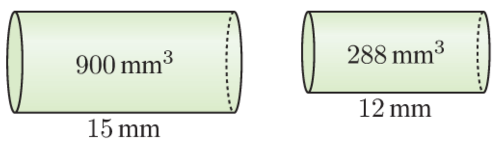


6 Determine whether each pair of figures below are similar:

a



b



7 A 2 m high statue has volume 1.5 m^3. If a similar model is only 40 cm high, what is its volume?

ANSWERS

EXERCISE 1A

1 **a** 23 **b** 18 **c** 78 **d** 168

2 **a** 503 **b** 3076 **3** €27

4 Yes, the total weight is only 1484 kg. **5** \$24 149

6 **a** 437 **b** 28 **c** 720

7 82 buckets of chips **8** 38 400 apples **9** 25 laps

10 **a** 840 apartments **b** \$2 856 000 **11** 63 plane loads

EXERCISE 1B

1 **a** 3^2 **b** 5^3 **c** 7^2 **d** 2^5
e $3^2 \times 7^3$ **f** 2×5^3 **g** $3^3 \times 5 \times 11^2$
h $2^4 \times 3^2 \times 7$ **i** $2^3 \times 5^2$

2 **a** 9 **b** 24 **c** 560 **d** 1300 **e** 550
f 360

EXERCISE 1C

1 **a** 1, 2, 4 **b** 1, 2, 3, 6 **c** 1, 3, 9
d 1, 3, 5, 15 **e** 1, 2, 4, 8, 16 **f** 1, 17
g 1, 2, 11, 22 **h** 1, 2, 3, 4, 6, 8, 12, 24
i 1, 2, 4, 7, 14, 28 **j** 1, 2, 3, 6, 7, 14, 21, 42
k 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
l 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108

2 **a** $36 = 6 \times 6$ **b** $38 = 2 \times 19$ **c** $48 = 12 \times 4$
d $90 = 5 \times 18$ **e** $88 = 8 \times 11$ **f** $54 = 3 \times 18$
g $72 = 12 \times 6$ **h** $60 = 12 \times 5$

3 **a** $1 \times 10,\ 2 \times 5$ **b** $1 \times 14,\ 2 \times 7$
c $1 \times 18,\ 2 \times 9,\ 3 \times 6$ **d** $1 \times 20,\ 2 \times 10,\ 4 \times 5$

4 **a** 6, 8, 10 **b** 11, 13, 15, 17, 19

5 **a** even **b** even **c** even **d** odd **e** odd
f odd **g** even **h** odd **i** even

EXERCISE 1D

1 **a** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59
b one, 2 **c** **i** 3 **ii** 15, 21, 25, 27 **iii** 29

2 **Note:** Other answers are possible.
a $985 = 5 \times 197$ **b** $7263 = 3 \times 2421$
c $5840 = 10 \times 584$ **d** $1001 = 11 \times 91$

3 **a** 2×7 **b** $2^2 \times 5$ **c** $2^2 \times 7$ **d** 2^5
e $2^3 \times 5$ **f** $2 \times 5 \times 7$ **g** $2^3 \times 7$ **h** $2^5 \times 3$
i $2^3 \times 3 \times 5$ **j** 13^2

EXERCISE 1E

1 **a** 4 **b** 3 **c** 7 **d** 9 **e** 13 **f** 6
g 3 **h** 4

2 8 friends

EXERCISE 1F

1 **a** 4, 8, 12, 16, 20, 24 **b** 5, 10, 15, 20, 25, 30
c 7, 14, 21, 28, 35, 42 **d** 11, 22, 33, 44, 55, 66

2 **a** 24 **b** 54

3 **a, b**

1	2	3	4	$\boxed{5}$	6	7	8	9	$\boxed{10}$
11	12	13	14	$\boxed{15}$	16	17	18	19	$\boxed{20}$
21	22	23	24	$\boxed{25}$	26	27	28	29	$\boxed{30}$
31	32	33	34	$\boxed{35}$	36	37	38	39	$\boxed{40}$

c 15, 30

4 **a** 36, 72, 108 **b** 60, 120
c none in the list (first one is 180)

5 **a** 10 **b** 21 **c** 20 **d** 24 **e** 18 **f** 60
g 140 **h** 36

6 **a** 105 **b** 198 **7** in 12 hours' time **8** 14.4 km

EXERCISE 1G

1 **a** 7 **b** 8 **c** 11 **d** 12

2 **a** 17 **b** 24 **c** 39 **d** 48

3 **a** 3 and 4 **b** 5 and 6 **c** 10 and 11 **d** 12 and 13

4 **a** 1 **b** -1 **c** -3 **d** 2 **e** -2 **f** -4
g 5 **h** 6 **i** -10 **j** 7

EXERCISE 1H

1 **a** 7 **b** 14 **c** 13 **d** 5 **e** 8 **f** 5
g 7 **h** 5 **i** 2 **j** 4 **k** 1 **l** 8

2 **a** 15 **b** 3 **c** 3 **d** 16 **e** 32 **f** 2
g 17 **h** 4 **i** 8 **j** 8 **k** 6 **l** 36
m 4 **n** 2 **o** 14

3 **a** 12 **b** 7 **c** 14 **d** 6 **e** 5 **f** 64

4 **a** 8 **b** $\frac{1}{4}$ **c** 3 **d** 2

5 **a** 28 **b** 31 **c** 13 **d** 1 **e** 12 **f** 38
g 23 **h** 1 **i** 7 **j** 1 **k** 6 **l** -1

6 **a** 12 **b** -12 **c** 38 **d** 2 **e** -12 **f** -6
g 30 **h** 3 **i** -11 **j** 4 **k** -1 **l** -2
m 5 **n** -4 **o** -1

EXERCISE 1I

1 **a** 3 **b** 11 **c** -7 **d** 16 **e** -6
f 16 **g** 18 **h** -2 **i** 2

2 **a** 2 **b** 2 **c** $\frac{1}{4}$ **d** $\frac{1}{4}$

EXERCISE 1J.1

1 **a** 90 **b** 80 **c** 90 **d** 130 **e** 160
f 100 **g** 640 **h** 1820 **i** 700 **j** 3050

2 **a** 200 **b** 300 **c** 3800 **d** 4000 **e** 26 300

3 **a** 8000 **b** 4000 **c** 19 000 **d** 20 000 **e** 115 000

4 **a** 8850 m **b** 32 000 km^2 **c** 4 750 000 people
d 85 500 people **e** 7 700 000 km^2 **f** 5400 kg
g 17 000 km **h** 400 000 km **i** 428 000 000 people

EXERCISE 1J.2

1 **a** 6.2 **b** 6.18 **c** 3.3 **d** 17.40
e 2.132 **f** 0.2 **g** 0.10 **h** 102.38

2 9.6 s **3** 1.44 m **4** 0.01 cm

5 **a** 3.1 **b** 3.142 **c** 3.1416

6 **a** 0.3 **b** 0.26 **c** 0.263 158

7 **a** ≈ 1.414 **b** ≈ 2.236 **c** ≈ 4.796 **d** ≈ 1.587
e ≈ -2.466 **f** ≈ 7.663

8 **a** 499.32 **b** 228.84 **c** 9.11 **d** 31.75
e 26.67 **f** 0.88 **g** 7.41 **h** 5.93
i 0.48

9 1.74 goals per game

10 While $2.45\text{ m} \approx 2.5\text{ m}$ is correct, Wang should have used the original value of 2.45 m to round to the nearest integer, so $2.45\text{ m} \approx 2\text{ m}$.

EXERCISE 1J.3

1 **a** 130 **b** 8300 **c** 2.6 **d** 0.013 **e** 160 000 **f** 1.1 **g** 4000 **h** 6.6

2 **a** 83 100 **b** 10 000 **c** 0.105 **d** 31.7 **e** 70.7 **f** 4.00 **g** 0.0367 **h** 20.0

3 **a** 16.38 **b** 438.2 **c** 6 874 000 **d** 0.028 89

4 **a** 100 000 people **b** 96 000 people **c** 96 300 people

5 **a** ≈ 2.65 **b** ≈ 6.28 **c** ≈ 2.12 **d** $\approx 1\,970\,000$ **e** ≈ 0.932 **f** ≈ 4.39 **g** ≈ 1.79 **h** ≈ 5.73

6 900 seats **7** 256 m^2 **8** **a** \$188 **b** \$188.06

EXERCISE 1K

1 **a** 180 **b** 420 **c** 2400 **d** 600 **e** 25 000 **f** 24 000 **g** 1000 **h** 2400 **i** 5000 **j** 60 000 **k** 120 000 **l** 120 000 **m** 3 **n** 3.5 **o** 180

2 **a** 30 **b** 12 **c** 25 **d** 15 **e** 200 **f** 1000 **g** 5 **h** 250 **i** 0.2 **j** 10 **k** 2000 **l** 5

3 **a** €24 **b** \$600 **c** \$630 **d** £60

4 **a** 240 km **b** 4000 days **c** 20 tonnes

EXERCISE 2A

1 **a** $5x$ **b** $2c$ **c** $7q$ **d** $4fg$ **e** $6pq$ **f** $9rs$ **g** $6ab$ **h** $4mn$ **i** $5ab$ **j** $2pq$ **k** jkl **l** dhp

2 **a** $pq+r$ **b** $4x+5y$ **c** $2a-b$ **d** $ab-c$ **e** $b-ac$ **f** $f-7g$ **g** $ac+ad$ **h** $12-6rs$ **i** $3(x+y)$ **j** $5(d-1)$ **k** $8(w-x)$ **l** $pq(r-2)$

3 **a** $2b$ **b** $3q$ **c** $2x+4y$ **d** $3c+e$ **e** $3+z+2y$ **f** $4a+7$ **g** $2+4g$ **h** $3-3d$ **i** s **j** $s-2t$ **k** $5+3r$ **l** $2+2a+2b$

4 **a** $a\times a\times a\times a$ **b** $f\times f$ **c** $5\times a\times a$ **d** $5\times a\times 5\times a$ **e** $4\times p\times p\times p$ **f** $4\times p\times 4\times p\times 4\times p$ **g** $3\times t\times t\times t\times t\times t$ **h** $5\times x\times x\times x\times y$ **i** $5\times x\times x\times y\times x\times y$ **j** $7\times f\times f\times g\times g\times g$ **k** $p\times p+2\times q$ **l** $p\times p\times p-3\times q\times q$

5 **a** $3k^2$ **b** $4a^3$ **c** $2d^4$ **d** $4pq^2$ **e** $3f^2g^2$ **f** w^2xy^3 **g** $m+m^2$ **h** n^3+n **i** y^2-z^4 **j** a^2+7a **k** $8b-b^3$ **l** $2pq^2+6r^2s$ **m** $2h^2-hj$ **n** $3x+5x^3$ **o** $a^2+2b^3-ab^2$

EXERCISE 2B

1 **a** $x+6$ **b** $q+11$ **c** $3b+3$ **d** $2a+7$ **e** $2d$ **f** $2q+5$ **g** $2y$ **h** $3z$ **i** $2g^2$ **j** cannot be simplified **k** w^2 **l** cannot be simplified **m** $2x$ **n** $9ab$ **o** $4m$

2 **a** 0 **b** $7p$ **c** cannot be simplified **d** $6pq$ **e** $4ab$ **f** $2q^2$ **g** $12w$ **h** $13xy$ **i** $3z$ **j** 0 **k** $9d-9$ **l** $7g-7g^2$ **m** $4s+4s^2$ **n** cannot be simplified **o** cannot be simplified

3 **a** $8x$ **b** $6x$ **c** $-8x$ **d** cannot be simplified **e** $5k-5$ **f** $-10n$ **g** $-15m$ **h** $-5j+4$ **i** $7y$ **j** $7y$ **k** $9y$ **l** $5y$

4 **a** $-2x-1$ **b** $5t+3$ **c** $-2x-y$ **d** $6pq-4$ **e** $10cd$ **f** $-a$ **g** $5x^2-2$ **h** $-3n-3$ **i** $-5v-5w$ **j** $-3x^2$ **k** $5a-7b$ **l** $-5z-7$ **m** $p-2pq$ **n** $-7mn-2m$

EXERCISE 2C

1 **a** $2x+8$ **b** $3x-3$ **c** $-2x-4$ **d** $-2+x$ **e** x^2+x **f** $8x+20$ **g** $4x-1$ **h** x^2-x **i** $3x$ **j** $6x+2$ **k** $16-6x$ **l** $22x+35$ **m** $2x^2-x$ **n** a^2-6 **o** $4a-3b$

2 **a** x^2-x-6 **b** x^2+3x-4 **c** x^2+5x+6 **d** $2x^2+5x+3$ **e** $3x^2+10x+8$ **f** $10x^2+x-2$ **g** $3x^2+x-10$ **h** $14+17x-6x^2$ **i** $5-13x-6x^2$ **j** $15x^2+11x-12$ **k** $2-11x+15x^2$ **l** $21-17x+2x^2$ **m** $15-16x+4x^2$ **n** $-x^2-3x-2$ **o** $-4x^2-2x+6$

3 **a** x^2-36 **b** x^2-64 **c** $4x^2-1$ **d** $9x^2-4$ **e** $16x^2-25$ **f** $25x^2-9$ **g** $9-x^2$ **h** $49-x^2$ **i** $49-4x^2$ **j** x^2-2 **k** x^2-5 **l** $4x^2-3$

4 **a** $x^2+10x+25$ **b** $x^2+14x+49$ **c** x^2-4x+4 **d** $x^2-12x+36$ **e** $9+6x+x^2$ **f** $25+10x+x^2$ **g** $121-22x+x^2$ **h** $100-20x+x^2$ **i** $4x^2+28x+49$ **j** $9x^2+12x+4$ **k** $25-20x+4x^2$ **l** $49-42x+9x^2$

EXERCISE 2D

1 **a** $\dfrac{x+15}{5}$ **b** $\dfrac{x+3}{x}$ **c** $\dfrac{x+4}{2}$ **d** $\dfrac{14-x}{4}$ **e** $\dfrac{8x-2}{15}$ **f** $\dfrac{4x+17}{12}$ **g** $\dfrac{x^2+4}{2x}$ **h** $\dfrac{x^2-x-3}{3x}$

2 **a** $\dfrac{x+5}{x+2}$ **b** $\dfrac{11-2x}{x-4}$ **c** $\dfrac{1-3x}{x-1}$ **d** $\dfrac{5x+2}{x+1}$ **e** $\dfrac{2x+3}{x+1}$ **f** $\dfrac{x+3}{1-x}$

3 **a** $\dfrac{5x+4}{x(x+2)}$ **b** $-\dfrac{1}{(x-2)(x-3)}$ **c** $-\dfrac{x+10}{(x+1)(x-2)}$ **d** $\dfrac{8x^2+6x+8}{(x+4)(x-4)}$ **e** $\dfrac{7x^2-6x-25}{(2x-5)(x-2)}$ **f** $\dfrac{3x^2+3x+13}{(2x+1)(x-3)}$

EXERCISE 2E

1 **a** $\dfrac{7x}{4y}$ **b** $\dfrac{a}{b}$ **c** $\dfrac{y}{4x}$ **d** $\dfrac{3m}{2}$ **e** $\frac{1}{3}$ **f** $\dfrac{30}{t^2}$ **g** $\dfrac{3x}{8y}$ **h** 1 **i** $\dfrac{16}{k^2}$ **j** $15a$ **k** $\dfrac{28m^2}{3n}$ **l** $\dfrac{5x^2}{4y}$

2 **a** $\dfrac{x^2}{6}$ **b** $\dfrac{4}{x^3}$ **c** b **d** $\frac{6}{5}$ **e** $\dfrac{x^2}{8}$ **f** $2a$ **g** $\dfrac{x^3}{3}$ **h** $\dfrac{6}{z}$ **i** $\dfrac{3}{y}$ **j** $\dfrac{b}{15}$ **k** $\dfrac{4}{x^3y}$ **l** $\dfrac{2b}{3a}$

EXERCISE 2F

1 **a** $2(x-3)$ **b** $3(x+1)$ **c** $2(3-x)$
d $x(x+2)$ **e** $x(3-x)$ **f** $3x(x+3)$
g $x(2x+7)$ **h** $2x(2x-5)$ **i** $3x(2x-5)$

2 **a** $(x+6)(x-6)$ **b** $(3x+5)(3x-5)$
c $(4x+1)(4x-1)$ **d** $2(x+2)(x-2)$
e $3(x+\sqrt{3})(x-\sqrt{3})$ **f** $4(x+\sqrt{5})(x-\sqrt{5})$

3 **a** $(x-4)^2$ **b** $(x-5)^2$ **c** $2(x-2)^2$
d $(4x+5)^2$ **e** $(3x+2)^2$ **f** $(x-11)^2$

4 **a** $(x+1)(x+8)$ **b** $(x+3)(x+4)$
c $(x+2)(x-9)$ **d** $(x-3)(x+7)$
e $(x-3)(x-6)$ **f** $(x-2)(x+3)$
g $-(x+1)(x-2)$ **h** $3(x-3)(x-11)$
i $-2(x+1)^2$ **j** $2(x-2)(x+5)$
k $2(x-8)(x+3)$ **l** $-2(x-1)(x-6)$
m $-3(x-1)^2$ **n** $-(x+1)^2$
o $-5(x+2)(x-4)$

5 **a** $(2x-3)(x+4)$ **b** $(3x+1)(x-2)$
c $(7x-2)(x-1)$ **d** $(3x-2)(2x+1)$
e $(2x-3)(2x+1)$ **f** $(5x-3)(2x+1)$
g $(2x+1)(x-6)$ **h** $(3x+7)(x-4)$
i $(4x+3)(2x-1)$ **j** $(5x+3)(2x-3)$
k $(3x-1)(x+8)$ **l** $(3x+2)(2x+1)$
m $-2(2x+3)(x-1)$ **n** $(6x+1)(2x-3)$
o $-3(2x+7)(x-2)$ **p** $-(3x-2)(3x-5)$
q $(4x-9)(2x+3)$ **r** $(4x+3)(3x+1)$
s $(6x+1)(2x+3)$ **t** $(5x-4)(3x-2)$
u $(7x+5)(2x-3)$

EXERCISE 2G

1 **a** 15 **b** 3 **c** -1 **d** -10
e -8 **f** 11 **g** 14 **h** -10

2 **a** 8 **b** 1 **c** 6 **d** 6
e 1 **f** -18 **g** -4 **h** -8

3 **a** $\frac{3}{2}$ **b** -1 **c** $\frac{1}{2}$ **d** $-\frac{3}{5}$

4 **a** $-\frac{3}{4}$ **b** -1 **c** $-\frac{3}{2}$ **d** -5

5 **a** 1 **b** -27 **c** 25 **d** 1
e 63 **f** 27 **g** 36 **h** 18

6 **a** 1 **b** 4 **c** 5 **d** $\sqrt{53}$
e $\sqrt{2}$ **f** $\sqrt{58}$

EXERCISE 2H

1 **a** $x=-2$ **b** $x=7$ **c** $x=6$ **d** $x=-4$
e $x=\frac{11}{2}$ **f** $x=-3$ **g** $x=7$ **h** $x=\frac{4}{3}$

2 **a** $x=45$ **b** $x=64$ **c** $x=-3$ **d** $x=35$

3 **a** $x=51$ **b** $x=1$ **c** $x=7$ **d** $x=\frac{23}{2}$
e $x=17$ **f** $x=14$

4 **a** $x=1$ **b** $x=-4$ **c** $x=9$ **d** $x=-2$
e $x=\frac{1}{2}$ **f** $x=6$

5 **a** $x=6$ **b** $x=3$ **c** $x=2$ **d** $x=-4$
e $x=\frac{1}{2}$ **f** $x=\frac{10}{3}$

6 **a** $x=1$ **b** $x=4$ **c** $x=-2$ **d** $x=-\frac{7}{10}$

7 The equation can be simplified to $-3=-3$, which is true for all values of x.

8 The equation can be simplified to $-2=6$, but there are no values of x for which this is true. So, the equation has no solutions.

9 **a** $x=\frac{10}{3}$ **b** $x=\frac{25}{6}$ **c** $x=\frac{17}{5}$ **d** $x=-4$
e $x=\frac{3}{4}$ **f** $x=\frac{7}{12}$

EXERCISE 2I

1 **a** $x>9$ **b** $x\leqslant 7$ **c** $x\geqslant -2$ **d** $x<-4$
e $x\leqslant 9$ **f** $x<-9$ **g** $x\geqslant 5$ **h** $x<-3$
i $x<6$

2 **a** $x\leqslant 7$ **b** $x>-6$ **c** $x\geqslant \frac{1}{2}$

3 **a** $x>-\frac{2}{3}$ **b** $x\leqslant -4$ **c** $x<-22$ **d** $x\geqslant \frac{9}{5}$
e $x<27$ **f** $x\geqslant 6$

EXERCISE 2J

1 **a** $x=\frac{15}{2}$ **b** $x=6$ **c** $x=\frac{28}{3}$ **d** $x=\frac{15}{8}$
e $x=\frac{7}{5}$ **f** $x=-\frac{13}{2}$

2 The equation can be simplified to $16=15$. But there are no values of x for which this is true. So, the equation has no solutions.

3 **a** $x=1$ **b** $x=4$ **c** $x=-\frac{10}{3}$ **d** $x=\frac{1}{5}$
e $x=-15$ **f** $x=21$

EXERCISE 2K

1 **a** $x=\pm 3$ **b** $x=\pm 7$ **c** $x=\pm 6$
d $x=0$ **e** $x=\pm 1$ **f** $x=\pm\sqrt{17}$
g $x=\pm\sqrt{23}$ **h** $x=\pm 10$ **i** no real solutions
j no real solutions **k** $x=\pm\sqrt{27}$ **l** no real solutions

2 **a** $x=2$ **b** $x=3$ **c** $x=0$
d $x=4$ **e** $x=-1$ **f** $x=-5$
g $x=\frac{1}{2}$ **h** $x=\sqrt[3]{36}$ (≈ 3.30)

3 **a** $x=\pm 2$ **b** $x=\pm 3$ **c** $x=\pm 5$
d $x=\pm 4$ **e** $x=\pm 6$ **f** $x=\pm 5$
g $x=\pm\sqrt{50}$ (or $\pm 5\sqrt{2}$) **h** $x=\pm\sqrt{44}$ (or $\pm 2\sqrt{11}$)
i $x=\pm\sqrt{12}$

4 **a** $x=1$ **b** $x=-2$ **c** $x=6$
d $x=100$ **e** $x=\sqrt[3]{16}$ (≈ 2.52)
f $x=-\sqrt[3]{800}$ (≈ -9.28)

EXERCISE 2L

1 **a** $P=23$ **b** $P=9$ **c** $P=0$ **d** $P=-26$

2 **a** ≈ 25.1 cm **b** ≈ 34.6 m **c** ≈ 49.3 cm

3 **a** 44.1 m **b** ≈ 129 m

4 **a** ≈ 5.03 km **b** ≈ 11.3 km **c** ≈ 15.9 km

5 **a** $80\ \text{km h}^{-1}$ **b** 260 km **c** ≈ 7 h 52 min

6 **a** 0.48 volts **b** 60 ohms

7 **a** $\approx 7920\ \text{cm}^3$ **b** ≈ 1.59 cm **c** ≈ 5.95 mm

EXERCISE 2M

1 **a** $y=\dfrac{4-x}{2}$ **b** $y=\dfrac{7-2x}{6}$ **c** $y=\dfrac{11-3x}{4}$
d $y=\dfrac{8-5x}{4}$ **e** $y=\dfrac{20-7x}{2}$ **f** $y=\dfrac{38-11x}{15}$

2 **a** $y = \dfrac{x-4}{2}$ **b** $y = \dfrac{2x-7}{6}$ **c** $y = \dfrac{3x+12}{4}$
d $y = \dfrac{4x-18}{5}$ **e** $y = \dfrac{7x-42}{6}$ **f** $y = \dfrac{12x+44}{13}$

3 **a** $x = b - a$ **b** $x = \dfrac{b}{a}$ **c** $x = \dfrac{d-a}{2}$
d $x = t - c$ **e** $x = \dfrac{d-3y}{7}$ **f** $x = \dfrac{c-by}{a}$
g $x = \dfrac{c+y}{m}$ **h** $x = \dfrac{c-p}{2}$ **i** $x = \dfrac{a-t}{3}$
j $x = \dfrac{n-5}{k}$ **k** $x = \dfrac{a-n}{b}$ **l** $x = \dfrac{a-p}{n}$

4 **a** $x = ab$ **b** $x = \dfrac{a}{d}$ **c** $x = \dfrac{2}{p}$
d $x = 2n$ **e** $x = \dfrac{5z}{y}$ **f** $x = \pm\sqrt{mn}$

5 $y = -\frac{5}{3}x + 6, \quad m = -\frac{5}{3}, \quad c = 6$

6 **a** $s = \dfrac{R-2t}{5}$ **b** **i** $s = 2$ **ii** $s = -4$ **iii** $s = \frac{12}{5}$

7 **a** $x = \dfrac{t}{2My}$ **b** **i** $x = \frac{1}{15}$ **ii** $x = \frac{5}{4}$

8 **a** $r = \sqrt{\dfrac{A}{\pi}} \quad \{r > 0\}$
b **i** ≈ 1.13 cm **ii** ≈ 1.99 cm **iii** ≈ 2.82 cm

9 **a** $d = st$
i 180 km **ii** 120 km **iii** $126\frac{2}{3}$ km
b $t = \dfrac{d}{s}$
i 3 hours **ii** 4 hours **iii** 2 h 12 min

EXERCISE 3A

1 **a** 82.5 m **b** 29.5 cm **c** 6.25 km
d 7.38 cm **e** 0.2463 m **f** 0.009 761 km

2 **a** 4130 mm **b** 3 754 000 m **c** 482 900 cm
d 26 900 mm **e** 47 000 cm **f** 3 880 000 mm

3 2.55 km **4** 2800 candles **5** 13 ells

6 **a** 0.056 m^2 **b** 0.55 cm^2 **c** 0.000 008 43 km^2
d 0.382 ha **e** 0.0721 km^2 **f** 9.89 m^2

7 **a** 210 mm^2 **b** 3800 cm^2 **c** 480 ha
d 59 m^2 **e** 832 500 m^2 **f** 130 000 m^2

8 ≈ 845 m^2 **9** 180 panels

10 **a** 39.1 m^3 **b** 510 mm^3 **c** 0.469 m^3
d 3 820 000 cm^3 **e** 0.005 27 cm^3 **f** 17 900 cm^3
g 692 cm^3 **h** 0.183 46 m^3 **i** 5100 cm^3

11 5 000 000 ball bearings **12** ≈ 0.216 m^3

13 **a** 4210 mL **b** 8630 L **c** 4.6 L
d 56.9 kL **e** 0.003 97 kL **f** 12 000 mL

14 40 755.2 L **15** 1860 bottles

16 **a** 5900 g **b** 2.6 kg **c** 0.003 75 t
d 15 000 000 mg **e** 4 800 000 g **f** 0.0016 kg
g 1738.5 mg **h** 0.046 218 t **i** 36 100 mg

17 20 000 nails **18** **a** 6.21 t **b** 4 truck loads

19 **a** 3.2 g **b** 17 karat

20 **a** 900 s **b** 2520 s **c** 4800 s
d 227 s **e** 11 178 s **f** 18 985 s

21 **a** 270 min **b** 34 min **c** 323 min

22 **a** 3 h 31 min **b** 6 h 5 min **c** 3 h 5 min
d 2 h 35 min

23 **a** 74.08 km h^{-1} **b** ≈ 10.8 kn **c** ≈ 15.3 m s^{-1}
d 108 km h^{-1} **e** ≈ 1.06 m s^{-1} **f** 4.464 km h^{-1}

24 71 km h^{-1} ≈ 19.7 m s^{-1} $\therefore$ 20.8 m s^{-1} is faster.

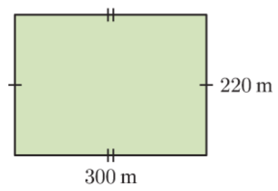
25 **a** 273.15 K **b** 27 kWh **c** 6.482 km

26 **a** 26.58 μs **b** 3200 nm **c** 2.53 GL
d 200 μg **e** 0.6 mL **f** 0.0685 mg

EXERCISE 3B

1 **a** 11.7 m **b** 35 mm **c** 6.2 km
d 172 cm **e** 26.8 cm **f** $(4x+2)$ m
g $(5x-2)$ km **h** $(6y-8)$ cm **i** $8x$ m

2 **a**



b 1040 m

3 **a** 132 m **b** 650 m **c** 66 posts **d** \$1097.70

4 **a** 19.2 m **b** \$19.84

5 **a** ≈ 25.13 cm **b** ≈ 16.34 m **c** ≈ 7.969 cm
d ≈ 38.05 m

6 **a** ≈ 10.7 m **b** ≈ 44.3 m **c** ≈ 410 mm

7 ≈ 1285 m **8** **a** 91.5 m **b** ≈ 156 m

9 **a** ≈ 11.9 m **b** ≈ 5.20 m **10** ≈ 1.27 m

EXERCISE 3C

1 **a** 9 m^2 **b** 23.6 cm^2 **c** 20 m^2
d 6 cm^2 **e** 24 cm^2 **f** 18 cm^2

2 **a** ≈ 113.1 m^2 **b** ≈ 63.6 cm^2 **c** ≈ 77.0 cm^2

3 **a** $(2x^2 + 9x + 4)$ cm^2 **b** πy^2 mm^2
c $\dfrac{5x^2+3x}{2}$ m^2 **d** $\dfrac{3x^2+3x}{2}$ cm^2
e $(y^2 + 2y)$ cm^2 **f** $2\pi x^2$ cm^2

4 **a** 83 cm^2 **b** 506 cm^2 **c** 143 m^2

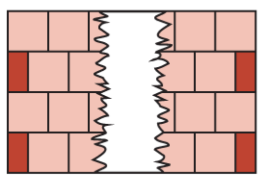
5 **a** 125 cm^2 **b** ≈ 76.0 cm^2 **c** 90 cm^2

6 **a** $A = 2k^2$ **b** $A = \dfrac{\pi(z^2 - y^2)}{2}$ **c** $A = (\pi - 2)x^2$

7 55 cm^2 **8** the medium pizza

9 **a** ≈ 48.6 m **b** 221 bricks **c** ≈ 147 m^2
d $\approx$ \$3250.20

10 **a** 43.2 m^2
b Size B slab is half the area of size A slab. Every 2nd row you need 2 size B slabs, so you need 4 size B slabs in total.



c 118 size A slabs **d** 2.6 tonnes **e** \$835.20

EXERCISE 3D

1 **a** ≈ 941 km h^{-1} **b** ≈ 27.1 km h^{-1}

2 **a** ≈ 8.20 m s^{-1} **b** ≈ 66.7 m s^{-1}

3 **a** ≈ 1 h 26 min **b** ≈ 42 min **c** ≈ 59 s
d ≈ 4 min 40 s

4 **a** 840 km **b** ≈ 17.2 km **c** 20.3 m
d 1440 m

5 **a** ≈ 274 km **b** ≈ 5.60 h **c** $\approx 48.9\ \text{km h}^{-1}$

EXERCISE 4A

1 **a** $\sqrt{34}$ cm **b** $\sqrt{20}$ cm **c** $\sqrt{72}$ cm
d $\sqrt{250}$ km **e** ≈ 21.5 cm **f** ≈ 8.38 cm

2 **a** 8 cm **b** $\sqrt{15}$ cm **c** $\sqrt{24}$ km
d ≈ 2.53 km **e** $\sqrt{3}$ m **f** ≈ 8.98 cm

3 **a** $x=\sqrt{7}$ **b** $x=\sqrt{5}$ **c** $x=\sqrt{8}$
d $x=\sqrt{\frac{13}{16}}$ **e** $x=1$ **f** $x=\frac{1}{2}$

4 **a** $x=\sqrt{48}$ **b** $x=\sqrt{13}$ **c** $x=\sqrt{3}$
d $x=\sqrt{7}$ **e** $x=\sqrt{3}$ **f** $x=1$

5 **a** $x=\sqrt{21},\ y=\sqrt{5}$ **b** $x=4,\ y=\sqrt{17}$
c $x=\sqrt{2},\ y=\sqrt{3}$

6 **a** $x=\sqrt{14}$ **b** $x=\sqrt{\frac{97}{4}}$ (≈ 4.92)

7 **a** $AB=\sqrt{10}$ cm **b** $AB=\sqrt{20}$ cm **c** $AB=\sqrt{33}$ m

8 **a** 30 m^2 **b** ≈ 5.66 m^2

EXERCISE 4B

1 **a** not right angled **b** right angled **c** not right angled
d not right angled **e** right angled **f** right angled

2 **a** right angle at A **b** right angle at B **c** right angle at C

EXERCISE 4C

1 **a** 10 cm **b** $\sqrt{18}$ m $(\approx 4.24$ m$)$ **2** 20 mm

3 **a** $\sqrt{96}$ mm $(\approx 9.80$ mm$)$ **b** $5\sqrt{96}$ mm^2 $(\approx 49.0$ mm$^2)$

4 10 m $\times$ 30 m **5** $\sqrt{5}$ cm **6** $\sqrt{162}$ cm $(\approx 12.7$ cm$)$

7 **a** $x=\sqrt{\frac{97}{4}}$ (≈ 4.92) **b** $x\approx 8.58$

8 **a** $2\sqrt{39}$ m $(\approx 12.5$ m$)$ **b** $10\sqrt{39}$ m^2 $(\approx 62.4$ m$^2)$

9 **a** $\sqrt{20}$ cm $(\approx 4.47$ cm$)$ **b** $2\sqrt{20}$ cm^2 $(\approx 17.9$ cm$^2)$

10 5 cm **11** 12 mm **12** **a** $90°$ **b** $a^2+b^2=c^2$

13 **a** $x=4$ **b** $x=5$ **c** $x=\sqrt{208}$ (≈ 14.4)

14 5 cm **15** 29 mm

16 **a** BC is a diameter. **b** BC is not a diameter.

EXERCISE 4D

1 **a** $x\approx 0.663$ **b** $x\approx 4.34$ **c** $x\approx 2.23$

2 **a** ≈ 4.54 m **b** ≈ 4.17 m **3** ≈ 17.5 km

4 ≈ 87.1 km **5** **a** ≈ 2.66 km **b** ≈ 16 min

6 **a** ≈ 92.4 cm **b** ≈ 1.85 m^2

7 **a** Jack 30 km, Will 38 km **b** ≈ 48.4 km

8 ≈ 6.97 cm **9** **a** ≈ 73.9 m **b** ≈ 30.9 m

10 **a** ≈ 44.0 km to A and ≈ 61.6 km to B **b** $\approx \$2\,480\,000$

11 ≈ 72 m **12** 6 cm **13** ≈ 33.1 m

14 ≈ 14.6 cm **15** ≈ 17.3 cm **16** ≈ 8.60 m **17** yes

18 ≈ 1.92 m **19** ≈ 53.4 m **20** ≈ 44.4 km

EXERCISE 5A

1 **a** $\sqrt{10}$ units **b** $\sqrt{13}$ units **c** $\sqrt{58}$ units **d** $\sqrt{20}$ units
e 2 units **f** $\sqrt{20}$ units **g** 5 units **h** $\sqrt{53}$ units

2 **a** $\sqrt{45}$ km $(\approx 6.71$ km$)$ **b** $\sqrt{97}$ km $(\approx 9.85$ km$)$
c 5 km **d** 10 km

3 **a** $AB=\sqrt{10}$, $BC=\sqrt{10}$, $AC=\sqrt{32}$
Triangle ABC is isosceles.
b $AB=\sqrt{73}$, $BC=\sqrt{50}$, $AC=\sqrt{17}$
Triangle ABC is scalene.
c $AB=2$, $BC=2$, $AC=2$
Triangle ABC is equilateral.
d $AB=\sqrt{28}$, $BC=\sqrt{48}$, $AC=\sqrt{28}$
Triangle ABC is isosceles.

4 **a** $AB=\sqrt{13}$, $BC=\sqrt{65}$, $AC=\sqrt{52}$
Triangle ABC is right angled at A.
b $AB=\sqrt{20}$, $BC=\sqrt{20}$, $AC=\sqrt{40}$
Triangle ABC is right angled at B.
c $AB=\sqrt{10}$, $BC=\sqrt{40}$, $AC=\sqrt{10}$
Triangle ABC is not right angled.
d $AB=\sqrt{85}$, $BC=\sqrt{17}$, $AC=\sqrt{68}$
Triangle ABC is right angled at C.

5 **a** $AB=\sqrt{50}$, $BC=\sqrt{50}$, $AC=\sqrt{180}$
Triangle ABC is isosceles.
b $AB=\sqrt{65}$, $BC=\sqrt{13}$, $AC=\sqrt{52}$
Triangle ABC is a scalene right angled triangle, with right angle at C.
c $AB=\sqrt{10}$, $BC=\sqrt{20}$, $AC=\sqrt{10}$
Triangle ABC is an isosceles right angled triangle, with right angle at A.
d $AB=\sqrt{12}$, $BC=\sqrt{12}$, $AC=\sqrt{12}$
Triangle ABC is equilateral.

EXERCISE 5B

1 **a** $(2, 3)$ **b** $(1, -3)$ **c** $(-2, 3)$ **d** $(1, 1)$
e $(3, 1)$ **f** $(0, -1)$ **g** $(0, \frac{1}{2})$ **h** $(3, -1)$

2 **a** $(3, 6)$ **b** $(\frac{5}{2}, 4)$ **c** $(1, 1\frac{1}{2})$ **d** $(3, 0)$
e $(\frac{1}{2}, 3)$ **f** $(-1, 1)$ **g** $(-\frac{1}{2}, -1\frac{1}{2})$ **h** $(-2\frac{1}{2}, 4)$

3 **a** $(3, -5)$ **b** $(-2, 3)$ **c** $(-1, 5)$ **d** $(4, -2)$
e $(4, -1)$ **f** $(3, -\frac{1}{2})$

4 **a** $(0, -9)$ **b** $(5, -5)$ **5** $(1, -5)$ **6** $(5, -2)$

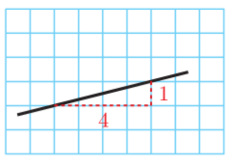
7 **a** $(-5, 4)$ **b** $\sqrt{10}$ km $(\approx 3.16$ km$)$ **8** $(0, 1)$

9 **a** $(9, -2)$ **b** $(3, 7)$ **c** $(0, -5)$

10 **a** **i** P$(2, 5)$ **ii** Q$(9, 4)$ **iii** R$(4, -1)$ **iv** S$(-3, 0)$
b **i** $\sqrt{50}$ units **ii** $\sqrt{50}$ units **iii** $\sqrt{50}$ units
iv $\sqrt{50}$ units
c PQRS is a rhombus.

EXERCISE 5C

1 **a** $\frac{3}{2}$ **b** -1 **c** 0 **d** $\frac{2}{5}$
e $-\frac{2}{3}$ **f** 3 **g** undefined **h** $-\frac{4}{3}$

2 **a**


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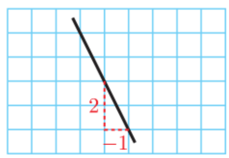

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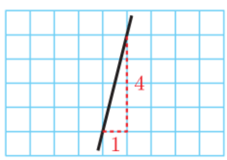

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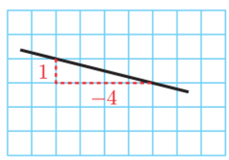
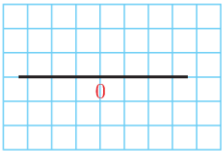
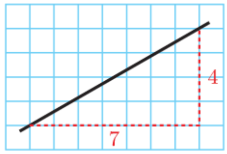
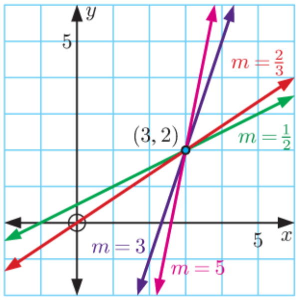
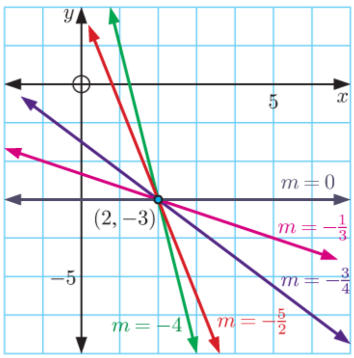


e 

f 

3 

4 

5 **a** $\frac{5}{3}$ **b** $-\frac{7}{3}$ **c** $-\frac{1}{3}$ **d** 0
e $-\frac{2}{3}$ **f** undefined

6 **a** $a=11$ **b** $a=6$ **c** $a=15$

7 Cuts the x-axis at $(4, 0)$.

EXERCISE 5D

1 **a** parallel, both lines have gradient $\frac{2}{5}$
b parallel, both lines have gradient $-\frac{1}{7}$
c perpendicular, the gradients of the lines are $\frac{1}{2}$ and $-\frac{2}{1}$, which are negative reciprocals of each other
d neither, the gradients of the lines are $\frac{1}{3}$ and $-\frac{4}{1}$
e neither, the gradients of the lines are $\frac{2}{7}$ and $\frac{1}{5}$
f perpendicular, the gradients of the lines are $\frac{2}{1}$ and $-\frac{1}{2}$, which are negative reciprocals of each other

2 **a** $-\frac{4}{3}$ **b** $-\frac{3}{11}$ **c** $-\frac{1}{4}$ **d** 3 **e** $\frac{1}{5}$
f undefined

3 **a** not perpendicular **b** not perpendicular
c perpendicular **d** not perpendicular
e perpendicular **f** not perpendicular

4 **a** $t=-3$ **b** $t=-10$

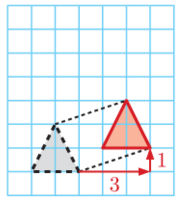
5 **a** $k=2$ **b** $k=1\frac{1}{2}$ **c** $k=7$ **d** $k=-\frac{10}{3}$

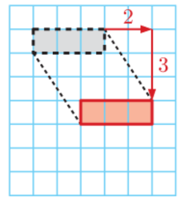
6 **a** $AB=\sqrt{20}$, $BC=\sqrt{5}$, $AC=5$
and $(\sqrt{20})^2+(\sqrt{5})^2=25=AC^2$
By Pythagoras, ABC is a right angled triangle.
b gradient of [AB] $=\frac{1}{2}$, gradient of [BC] $=-2$
$m_{[AB]}\times m_{[BC]}=-1$
$\therefore$ [AB] is perpendicular to [BC], ABC is a right angled triangle.

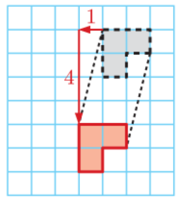
7 **a** collinear **b** not collinear **c** collinear
d not collinear

8 **a** $n=7$ **b** $n=0$

EXERCISE 6A

1 **a** 

b 

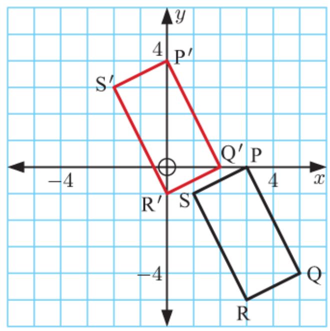
c 

2 **a** $\begin{pmatrix}3\\2\end{pmatrix}$ **b** $\begin{pmatrix}4\\-1\end{pmatrix}$ **c** $\begin{pmatrix}3\\-3\end{pmatrix}$ **d** $\begin{pmatrix}-3\\-5\end{pmatrix}$

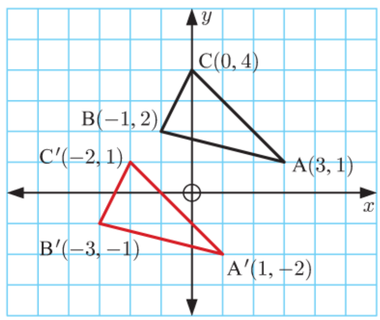
3 **a** $(3, 5)$ **b** $(1, 1)$

4 **a** $\begin{pmatrix}4\\3\end{pmatrix}$ **b** $\begin{pmatrix}-4\\2\end{pmatrix}$

5 **a** $(2, 2)$ **b** $(1, 3)$

6 **a** $P(3, 0)$, $Q(5, -4)$, $R(3, -5)$, $S(1, -1)$
b 

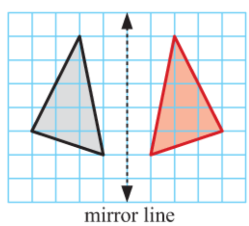
c $P'(0, 4)$, $Q'(2, 0)$, $R'(0, -1)$, $S'(-2, 3)$

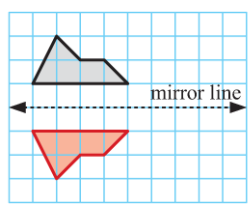
7 **a, b** 

c $A'(1, -2)$, $B'(-3, -1)$, $C'(-2, 1)$ **d** $\sqrt{13}$ units

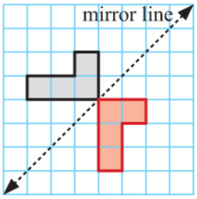
8 **a** a translation of $\begin{pmatrix}2\\3\end{pmatrix}$ **b** a translation of $\begin{pmatrix}5\\4\end{pmatrix}$

EXERCISE 6B

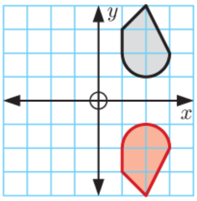
1 **a** 

b 

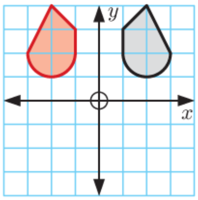
c



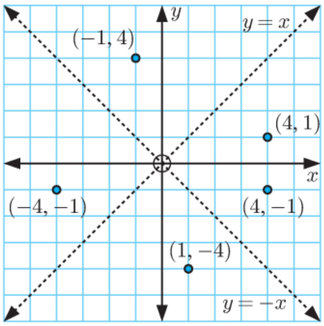
2 **a**



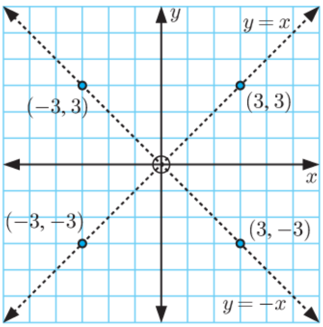
b



3 **a** (4, 1) **b** (−4, −1) **c** (−1, 4) **d** (1, −4)

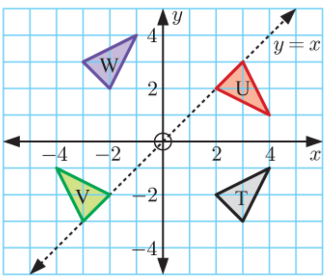


4 **a** (−3, 3) **b** (3, −3) **c** (−3, −3) **d** (3, 3)



(−3, −3) maps onto itself when reflected in the line $y = x$.

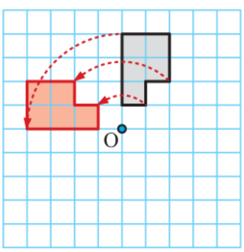
5



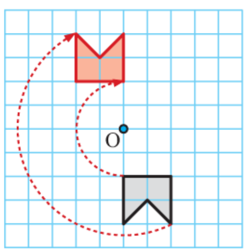
6 **a** (−1, −4)
b (0, −3)
c (1, −5)

EXERCISE 6C

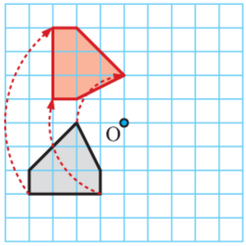
1 **a**



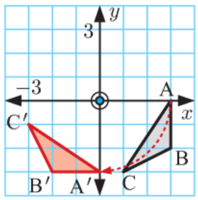
b



c



2 **a**

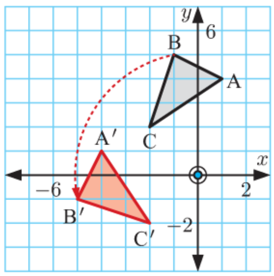


b A′(0, −3), B′(−2, −3), C′(−3, −1)

3 **a** (−3, −2) **b** (3, 2) **c** (2, −3)

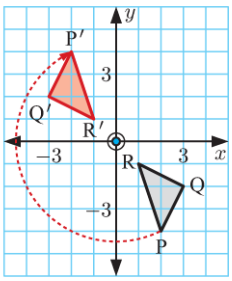
4 **a** (1, 4) **b** (−1, −4) **c** (−4, 1)

5 **a**



b A′(−4, 1), B′(−5, −1), C′(−2, −2)

6 **a**

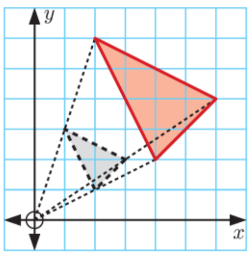


b P′(−2, 4), Q′(−3, 2), R′(−1, 1)

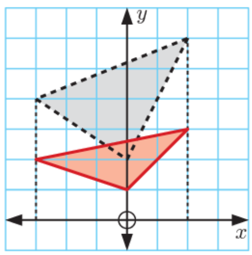
7 **a** (1, 4) **b** (1, 1) **c** (3, −1)

EXERCISE 6D

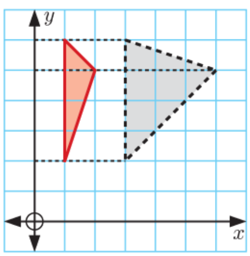
1 **a**



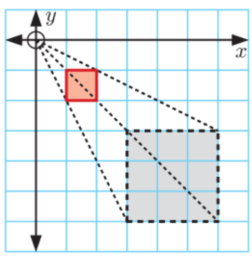
b



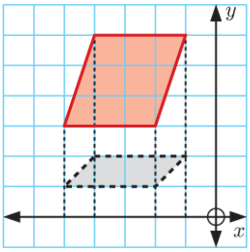
c



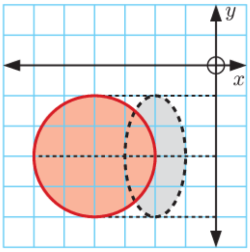
d



e

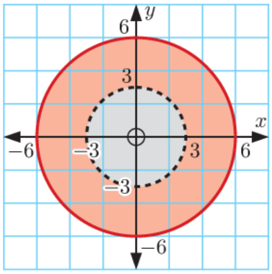


f

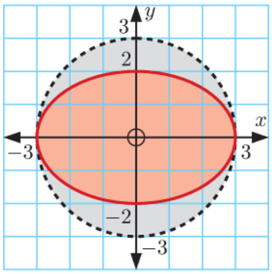


2 **a** $(6, -2)$ **b** $(-2, 4)$ **c** $(-2, -9)$
d $(1, 3)$ **e** $(-3, 3)$ **f** $(6, 2)$

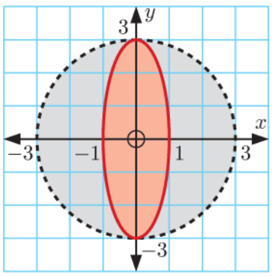
3 **a**



b



c



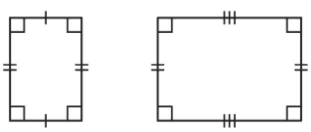
4 **a** An enlargement with centre O and scale factor 2.
b A vertical stretch with scale factor $\frac{1}{2}$.
c A horizontal stretch with scale factor 4.
d A vertical stretch with scale factor $\frac{5}{2}$.
e A reduction with centre O and scale factor $\frac{1}{3}$.
f A horizontal stretch with scale factor $\frac{3}{4}$.

EXERCISE 7A

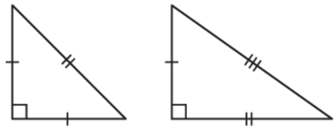
1 **a** similar **b** $\dfrac{A'B'}{AB} = \dfrac{3}{2}$, $\dfrac{A'C'}{AC} = 2$ $\therefore$ not similar.
c similar **d** similar

2 not similar

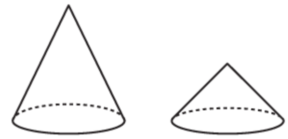
3 **a** true **b** false



c true **d** false



e true **f** false

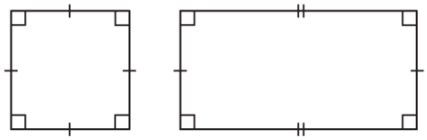


4 **a** $x = 5$ **b** $x = 6$ **c** $x = 6$ **d** $x = 127$

5 FG $= 2.925$ m

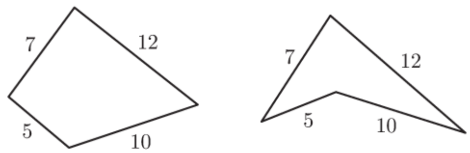
6 **a** $x = \frac{84}{5}$ **b** $x = 15$ **c** $x = 40$ **d** $x = \frac{21}{5}$

7 **a**



Note: Other answers are possible.

b



EXERCISE 7B

1 **a** $\widehat{ACB} = \widehat{DCE}$ {vertically opposite},
$\widehat{BAC} = \widehat{EDC}$ {angles in a triangle}
b $\widehat{FGH} = \widehat{FDE}$, angle F is common.
c $\widehat{BAC} = \widehat{DEC}$, $\widehat{ACB} = \widehat{ECD}$ {vertically opposite}
d $\widehat{TUV} = \widehat{TXW}$, angle T is common.
e $\widehat{BAC} = \widehat{DCA}$ {equal alternate angles},
$\widehat{ABC} = \widehat{ADC}$ {opposite angles of a parallelogram}
f $\widehat{ADO} = \widehat{CBA}$ {radius-tangent theorem},
angle A is common.

2 **a** $x = \frac{15}{2}$ **b** $x = 20$ **c** $x = 7$ **d** $x = \frac{24}{5}$
e $x = \frac{48}{7}$ **f** $x = 5\sqrt{2}$ **g** $x = \frac{15}{4}$

EXERCISE 7C

1 ≈ 1.80 m

2 **a** ≈ 7.00 m
b Stevie can stand further away from the shadow and still be able to use similar triangles to find the height of the building. However, if she stands inside the shadow, the ruler will not cast a shadow that can be measured.

3 42.5 m **4** $\sqrt{50}$ m ≈ 7.07 m **5** ≈ 7.92 m

6 ≈ 2.77 m

EXERCISE 7D

1 **a** 162 mm^2 **b** 18 cm^2 **c** 144 cm^2

2 **a** 14 cm **b** 16 mm **c** $\frac{54}{7}$ m

3 **a** $x = 640$ **b** $x = 8$ **c** $x = 625$ **d** $x = 8$

4 **a** $\widehat{ABC} = \widehat{CAD}$, $\widehat{BAC} = \widehat{ACD}$ {equal alternate angles}
b $k = \frac{7}{3}$ **c** **i** ≈ 217 cm^2 **ii** ≈ 257 cm^2

5 1.6 units^2 **6** **a** not similar **b** not similar

7 0.012 m^3